Elliptic Bessel Process and Elliptic Dyson Model Realized as Temporally Inhomogeneous Processes

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The Bessel process with the dimensions $D > 1$ and the Dyson model of interacting Brownian motions with the coupling constant $\beta > 0$ are extended to the processes, in which the drift term and the interaction terms are given by the logarithmic derivatives of Jacobi's theta functions. They are called the elliptic Bessel process, eBES$^{(D)}$, and the elliptic Dyson model, eDYS$^{(\beta)}$, respectively. Both are realized as temporally inhomogeneous processes defined in a finite time interval. The transformations of them to the quantum Calogero-Moser-Sutherland models with time-dependent potentials lead us to proving that eBES$^{(D)}$ and eDYS$^{(\beta)}$ can be constructed as the time-dependent Girsanov transformations of Brownian motions. In the special cases where $D = 3$ and $\beta = 2$, these processes are represented by the pinned processes with signed measures. We prove that eDYS$^{(2)}$ has the determinantal martingale representation. Then it is proved that eDYS$^{(2)}$ is determinantal for any finite initial configuration, in the sense that all spatio-temporal correlation functions are given by determinants controlled by a single continuous function called the correlation kernel. We will also discuss eDYS$^{(2)}$ with an infinite number of particles.