# Bessel Process, Schramm-Loewner Evolution, and Dyson Model

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The purpose of my lecture is to introduce recent topics in mathematical physics and probability theory, especially the topics on the Schramm-Loewner evolution (SLE) and interacting particle systems related with random matrix theory. A typical example of the latter systems is Dyson's Brownian motion model. For this purpose I have considered one story to tell the SLE and the Dyson model as 'children' of the Bessel processes [1]. The Bessel processes make a one-parameter family of one-dimensional diffusion processes with parameter $D$, in which the $D$-dimensional Bessel process, $\mathrm{BES}^{(D)}$, is defined as the radial part of the $D$-dimensional Brownian motion, if $D$ is an integer. This definition implies that Bessel processes are 'children' of the Brownian motion, and hence, the SLE and the Dyson model are 'grandchildren' of the Brownian motion.

First the parenthood of Brownian motion in diffusion processes is clarified and $\mathrm{BES}^{(D)}$ is defined for any $D \geq 1$. There, the importance of two aspects of $\mathrm{BES}^{(3)}$ is explained. SLE is then introduced as a complexification of $\mathrm{BES}^{(D)}$. I show that rich mathematics related with the conformal field theory and the fractal physics involved in SLE are due to the nontrivial dependence of the Bessel flow on $D$. Finally Dyson's Brownian motion model with parameter $\beta$ is introduced as a multivariate extension of $\mathrm{BES}^{(D)}$ with the relation $D = \beta + 1$. I will concentrate on the case where $\beta = 2$. In this case the Dyson model inherits the two aspects of $\mathrm{BES}^{(3)}$ and has very strong solvability. That is, the process is proved to be determinantal in the sense that all spatio-temporal correlation functions are given by determinants, and all of them are controlled by a single function called the correlation kernel.

[1] Katori, M.: *Bessel Process, Schramm-Loewner Evolution, and Dyson Model*, to be published in the series *SpringerBriefs in Mathematical Physics*, Springer (2015+).