Random Operators and Related Topics : Abstract

2023/10/13(Fri.) ∼ 10/15(Sun.), 10/16(Free Discussion)
Aoba Science Hall, Tohoku University

10/13 (Fri.)

10:00-10:50 **Makoto Katori (Chuo)**

Non-Hermitian matrix-valued processes and the regularized Fuglede-Kadison determinant random-fields

We consider a one-parameter family of non-Hermitian matrix-valued processes, which can be regarded as a dynamical extension of Girko's ensemble interpolating the GUE and the Ginibre ensemble of random matrices. For each process in this family, the biorthogonality relation is imposed between the right and the left eigenvector processes, which allows for the scale-transformation invariance of the system. There each eigenvalue process is coupled with the eigenvector-overlap process, which is a Hermitian matrix-valued process with entries given by products of overlaps of the right and left eigenvectors. The Fuglede–Kadison (FK) determinants of the present matrix-valued processes are regularized by introducing an auxiliary complex variable. Then, associated with the regularized FK determinants, the time-dependent random fields are defined in the two-dimensional complex space and their stochastic partial differential equations (SPDEs) are derived. Time-dependent point processes of eigenvalues and their variations weighted by the diagonal elements of the eigenvector-overlap processes are related to the logarithmic derivatives of the regularized FK-determinant random-fields. We also discuss the PDEs obtained by averaging the SPDEs. The present talk is based on the joint work with Yuya Tanaka, Saori Morimoto, Ayana Ezoe (Chuo), Syota Esaki (Fukuoka) and Satoshi Yabuoku (Kitakyushu).

11:00-11:50 **Khanh Duy Trinh (Waseda)**

Associated Laguerre polynomials in random matrix theory

Model I of associated Laguerre polynomials was introduced by Askey and Wimp (1984) by shifting the indices of coefficients in the three-term recurrence relations of (generalized) Laguerre polynomials by a real number. Model II, which is slightly different from Model I, was studied by Ismail, Letessier and Valent (1988) in relation with linear birth and death processes. In random matrix theory, the two models appear in the study of the global limiting behavior of beta Laguerre ensembles in a high temperature regime (Trinh and Trinh (2021) and Nakano, Trinh and Trinh (2023)). This talk introduces properties of the two models and related random matrix models.

11:50-13:00 Lunch

13:00-13:50 **Shu Nakamura (Gakushuin)**

Remarks on continuum limits for Schrödinger and Dirac operators

We discuss the continuum limits for Schrödinger operator on general (usual) lattices and the hexagonal lattice, and show the generalized norm resolvent convergence. We then discuss the continuum limits for more general operators corresponding to elliptic operators with variable coefficients. We also discuss the continuum limits for discrete Dirac operators, of which the definition is not obvious. The talk is based on joint works with Keita Mikami (Riken) and Yukihide Tadano (SUT).

14:00-14:50 **Yukimi Goto (Kyushu)**

Nambu-Goldstone modes in a lattice Nambu-Jona-Lasinio model with multi flavor symmetries

In this talk, we consider a lattice Nambu–Jona-Lasinio model with SU(2) and SU(3) flavor symmetries of staggered fermions in the Kogut–Susskind Hamiltonian formalism. This type of four-fermion interactions has been widely used for describing low-energy behaviors of strongly interacting quarks as an effective model. In the strong coupling regime for the interactions, we prove the following: (i) For the spatial dimension $d \geqq 5$, the SU(3) model shows a long-range order at sufficiently low temperatures. (ii) In the case of the SU(2) symmetry, there appears a long-range order in the spatial dimension $d \geqq 3$ at sufficiently low temperatures. (iii) These results hold in the ground states as well. (iv) In general, if a long-range order emerges in this type of models, then there exists a gapless excitation above an infinite-volume ground state. This is nothing but the Nambu–Goldstone mode associated with the spontaneous breakdown of the global rotational symmetry of flavors. This talk is based on joint work with Tohru Koma.

14:50-15:10 Coffee Break

15:10-16:00 **Naomasa Ueki (Kyoto)**

A definition of self-adjoint operators derived from the Schrödinger operator with the white noise potential on the plane

For the white noise ξ on $\mathbb{R}^2$, an operator corresponding to a limit of $-\Delta + \xi_\varepsilon + c_\varepsilon$ as $\varepsilon \to 0$ is realized as a self-adjoint operator, where, for each $\varepsilon > 0$, c_ε is a constant, ξ_ε is a smooth approximation of ξ defined by $\exp(\varepsilon^2\Delta)\xi$, and Δ is the Laplacian. This result is a variant of result's obtained by Allez and Chouk, Mouzard, and Ugurcan. The proof in this paper is based on the heat semigroup approach of the paracontrolled calculus, referring the proof by Mouzard. For the obtained operator, the spectral set is shown to be $\mathbb{R}$.

16:10-17:00 **Shinichi Kotani (Osaka, Emeritus)**

1D Schrödinger operator with whise noise potential and KdV equation

The Gaussian white noise is known to give a formal invariant measure to the Kdv equation, and its rigorous proof has been expected. The main difficulty is to prove the existence of a solution to the KdV equation. In 2020, R. Killip, J. Murphy, and M. Visan published a paper treating the solvability of the KdV equation with white noise initial data by using the invariance. On the other hand, the present speaker constructed a KdV flow on a general family of initial data including ergodic processes with some smootheness. The aim of this talk is to present the possibility of the second approach to the construction of a solution to the KdV equation starting from white noise. The main part is to obtain some asymptotic behavior of the Weyl function near the real line.

10/14(Sat.)

10:00-10:50 **Frédéric Klopp (Sorbonne)**

The ground state of a system of interacting fermions in a random field : localization, entanglement entropy,. . .

Transport in disordered solids is a phenomenon involving many actors. The motion of a single quantum particle in such a solid is described by a random Hamiltonian. Transport involves many interacting particles, usually, a small fraction of the particles present in the material. One striking phenomenon observed and proved in disordered materials is localization: disorder can prevent transport! While this is quite well understood at the level of a single particle, it is much less clear what happens in the case of many interacting particles. Physicist proposed a number of tools (exponential decay of finite particle density matrices, entanglement entropy, etc) to discriminate between transport and localization. Unfortunately, these quantities are very difficult to control mathematically for "real life" models. We'll present a toy model where one can actually get a control on various of these quantities at least for the ground state of the system. The talk is based on the PhD theses of and joint work with N. Veniaminov and V. Ognov.

11:00-11:50 **Constanza Rojas-Molina (Cergy)**

The long-range Anderson model with weakly decaying off-diagonal terms

We review results on the localization properties of long-range Anderson models in the discrete setting. This includes the fractional Anderson model, a long-range operator with polynomially decaying off-diagonal terms, and diagonal randomness. We discuss the effects of non-locality on the localization properties of the model, in particular, on dynamical localization. The interest in these models lies in their association to stable Levy processes, random walks with long jumps and anomalous diffusion.

11:50-13:00 Lunch

13:00-13:50 **Ryoki Fukushima (Tsukuba)**

Distribution of the random walk conditioned on survival among quenched Bernoulli obstacles

Consider a simple symmetric random walk that is conditioned to stay on a supercritical percolation cluster up to a large time n. Following a series of works of Sznitman in 1990s, it has recently been shown by Ding and Xu that with high probability, the random walk will be localized in a ball of volume proportional to log n. In this talk, I present the further refinements: (1) this ball is free of obstacles, (2) the limiting one-time distributions of the random walk are obtained. This talk is based on a joint work with Jian Ding, Rongfeng Sun and Changji Xu.

14:00-14:50 **Stefan Junk (Gakushuin)**

TBA

14:50-15:10 Coffee Break

15:10-16:00 **Takashi Imamura (Chiba)**

KPZ models and free fermions

Recently there has been much progress on our understanding of the one-dimensional Kardar-Parisi-Zhang (KPZ) universality class. For quite a few models in the KPZ class, Laplace-like transforms of the probability distributions of some observables can be expressed as Fredholm determinants or Pfaffians. Conventional approaches in integrable probability require somewhat involved calculations to get the final determinantal or Pfaffian formulas.

In this talk, I will report on our new approach to the KPZ models. The fundamental relations are combinatorial identities about two symmetric polynomials, the q-Whittaker polynomials and skew Schur polynomials, which can be regarded as connecting directly the KPZ models and free fermions. The Fredholm determinants or Pfaffians in the formulas are obtained immediately from the free fermionic side of the relations. This talk is based on collaboration with Matteo Mucciconi and Tomohiro Sasamoto.

[1] Takashi Imamura, Matteo Mucciconi, Tomohiro Sasamoto, Identity between restricted Cauchy sums for the q-Whittaker and skew Schur polynomials, arXiv:2106.11913.
[2] Takashi Imamura, Matteo Mucciconi, Tomohiro Sasamoto, Skew RSK dynamics: Greene invariants, affine crystals and applications to q-Whittaker polynomials, arXiv:2106.11922.
[3] Takashi Imamura, Matteo Mucciconi, Tomohiro Sasamoto, Solvable models in the KPZ class: approach through periodic and free boundary Schur measures, arXiv: 2204.08420.

16:10-17:00 **Tomohiro Sasamoto (Titech, online)**

TBA

18:00-20:00 Banquet, Kita-Aobayama Commons, I01 in the campus map

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10/15(Sun.)

10:00-10:50 **Tomi Ohtsuki (Sophia)**

Unifying the Anderson Transitions in Hermitian and Non-Hermitian Systems

The Anderson transition (AT) is a ubiquitous phenomenon observed in random systems, where waves undergo a localization-delocalization transition. In the case of Hermitian Hamiltonians, this transition is classified by the 10-fold Altland-Zirnbauer (AZ) symmetry classes. Non-Hermiticity enriches the AZ symmetry classes into the 38-fold symmetry classes [1], leading to changes in the critical behavior of the Anderson transitions, as demonstrated by finite size scaling analyses of the energy level statistics and localization length [2].

In this work, we propose a correspondence between the universality classes of the Anderson transitions in Hermitian and non-Hermitian systems [3,4]. Our analysis shows that critical exponents of length scales in non-Hermitian systems are identical to those in the corresponding Hermitian systems. This correspondence leads to a remarkable consequence of superuniversality, where the ATs in different symmetry classes of non-Hermitian systems are characterized by the same critical exponent. We compare the known critical exponents for non-Hermitian systems with their Hermitian counterparts, and numerically estimate the critical exponents in several symmetry classes in two and three dimensions, which are consistent with the proposed correspondence. Our correspondence explains why the magnon-Hall effect, described by the pseudo-Hermitian Hamiltonian, shows the same critical exponent as the quantum Hall effect in Hermitian systems.

References:

1. K. Kawabata, K. Shiozaki, M. Ueda, and M. Sato, Phys. Rev. X 9, 041015 (2019).
2. X. Luo, T. Ohtsuki, and R. Shindou, Phys. Rev. Lett. 126, 090402 (2021), Phys. Rev. B 104, 104203 (2021).
3. X. Luo, Z. Xiao, K. Kawabata, T. Ohtsuki, R. Shindou, Phys. Rev. Res. 4, L022035 (2022).
4. Z. Xiao, K. Kawabata, X. Luo, T. Ohtsuki, R. Shindou, Phys. Rev. Lett. 131, 056301 (2023).

11:00-11:50 **Keith Slevin (Osaka)**

A Stochastic Method to Compute the L^2 Localisation Landscape

The L2 localisation landscape of L. Herviou and J. H. Bardarson [1] is a generalisation of the localisation landscape of M. Filoche and S. Mayboroda [2]. In [3] we proposed a stochastic method to compute the L2 localisation landscape that enables the calculation of landscapes using sparse matrix methods. We also proposed an energy filtering of the L2 landscape which can be used to focus on eigenstates with energies in any chosen range of the energy spectrum. We demonstrated the utility of these suggestions by applying the L2 landscape to Anderson's model of localisation in one and two dimensions, and to localisation in a model of the quantum Hall effect.

1. Herviou, L. and J.H. Bardarson, L^2 localization landscape for highly excited states. Physical Review B, 2020. **101**(22): p. 220201.
2. Filoche, M. and S. Mayboroda, Universal mechanism for Anderson and weak localization. Proceedings of the National Academy of Sciences, 2012. **109**(37): p. 14761-14766.
3. Kakoi, M. and K. Slevin, A Stochastic Method to Compute the L2 Localisation Landscape. Journal of the Physical Society of Japan, 2023. **92**(5): p. 054707.

11:50-13:00 Lunch

13:00-13:50 **Taro Nagao (Nagoya)**

Correlation Functions of Two-dimensional Coulomb Gases on an Ellipse

The correlation functions of a class of two-dimensional Coulomb gases at a special temperature are known to be written in determinant forms. For that purpose, two-dimensional orthogonal polynomials on the complex plane are used. In this talk, the special case in which the Coulomb gases are on an ellipse is discussed.

14:00-14:50 **Hosho Katsura (Tokyo)**

Jordan blocks of non-Hermitian quadratic Hamiltonians

While Hermitian Hamiltonians can always be diagonalized by a unitary transformation, non-Hermitian Hamiltonians may exhibit exceptional points where they are non-diagonalizable and have nontrivial Jordan blocks. Determining the number and sizes of the Jordan blocks of such a Hamiltonian is difficult even for quadratic (non-interacting) systems. In this talk, I will discuss the Jordan decomposition of non-Hermitian fermionic quadratic forms such as the Hatano-Nelson model and a tight-binding model on a Cayley tree with complex potentials. I will also touch on our recent work [1] concerning Prosen's conjecture on the nilpotent part of the Jordan decomposition of a quadratic fermionic Liouvillian [2].

[1] S. Kitahama, H. Yoshida, R. Toyota, and H. Katsura, arXiv:2308.01166 (2023).
[2] T. Prosen, J. Stat. Mech. 2010, P07020 (2010).

14:50-15:10 Coffee Break

15:10-16:00 **Dimi Culcer (New South-Wales)**

Quantum kinetic theory of the orbital magnetic moment of Bloch electrons

The orbital magnetic moment (OMM) of Bloch electrons has been known for a long time for over half a century, and a well established semiclassical description of it exists. It has come under renewed scrutiny recently as part of a general effort to understand angular momentum dynamics in systems in which spin-orbit interactions are absent or negligible - including graphene, transition metal dichalcogenides, and topological antiferromagnets. Yet despite intense interest in the OMM its fundamental properties are poorly understood. At present there is no quantum mechanical theory of the OMM, part of the problem being that dealing with the position operator between Bloch states is non-trivial. This is a significant gap: without knowing when the OMM is conserved, for example, we cannot discuss meaningfully orbital currents and the orbital Hall effect. In this talk I will introduce a quantum mechanical theory of the OMM and its rate of change due to intrinsic mechanisms. The theory is based on the density matrix and quantum Liouville equation. I will show that, in an electric field, the force moment produces three contributions to the torque, of which two cancel, while the remaining contribution vanishes in two-band systems with particle-hole symmetry but is nonzero in general. I will discuss the torque in various Dirac fermion systems, as well as strategies for experimental verification. Our theory suggests that the torque on the OMM couples to the parameter that controls the inversion symmetry breaking of the system.

16:10-17:00 **Simone Warzel (TUM, Munich, online)**

Recent results in quantum spin glasses

We present recent results on Sherrington-Kirkpatrick model in a transversal magnet field. This includes a presentation of a variational expression for its free energy, a discussion of the order parameter and some results on the shape of the phase diagram of this model.

10/16(Mon.)

Free Discussion