Symmetries and Structures of Matrix-Valued Stochastic Processes and Noncolliding Diffusion Processes

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Part 4

Conférence aux Houches

"Vicious walkers and random matrices"

du 16 au 27 mai 2011

4^{th} day: Complex BM Representation and Eynard-Mehta-type Correlation Kernel

4.1 *h*-transform of Karlin-McGregor-LGV determinant

- Space of integer-valued Radon measures

$$\mathfrak{M} = \Big\{ \xi(\cdot) = \sum_{j \in \mathbb{I}} \delta_j(\cdot) : \quad \mathbb{I} \text{ is a countable index set,} \\ \xi(A) \equiv \sharp\{x_j : x_j \in A\} < \infty \ {}^{\forall}\text{compact set } A \subset \mathbb{R} \Big\}$$

$$\begin{aligned} \mathbb{I}(\mathbb{R}) = \sharp\mathbb{I} &< \infty, && \text{for finite particle systems} \\ &= \infty, && \text{for infinite particle systems} \end{aligned}$$

$$\mathfrak{M}_0 = \Big\{ \xi \in \mathfrak{M} : \xi(\{x\}) \leq 1, \ {}^{\forall}x \in \mathbb{R} \Big\}$$

no multiple point

- The Dyson model ($\beta = 2$ Dyson's BM model)

$$j \in \mathbb{I}$$

$$dX_j(t) = dB_j(t) + \sum_{k \in \mathbb{I}: k \neq j} \frac{dt}{X_j(t) - X_k(t)}, \quad t \in [0, \infty),$$

$$\{B_j(t)\}_{j \in \mathbb{I}} : \text{indep. one-dim. stand. BMs}$$

Regard the Dyson model as an $\mathfrak{M}$-valued process

$$\Xi(t, \cdot) = \sum_{j \in \mathbb{I}} \delta_{X_j(t)}(\cdot), \quad t \in [0, \infty)$$

with an initial configuration

$$\xi \in \mathfrak{M}, \quad \xi(\mathbb{R}) = \sharp \mathbb{I} \in \mathbb{N} \equiv \{1, 2, \dots\}.$$

- Notations

$\mathbb{P}_\xi$: probability law of $\xi(t, \cdot)$ starting from ξ

$\mathbb{E}_\xi[\,\cdot\,]$: expectation w.r.t. $\mathbb{P}_\xi$

$\{\mathcal{F}(t)\}_{t \in [0,\infty)}$: filteration

$\mathcal{F}(t) = \sigma(\Xi(s), s \in [0, t])$ on $C([0, \infty) \to \mathfrak{M})$ continuous path space

- As shown in my lectures

$$
\begin{aligned}
\text{the Dyson model} \quad &= \quad \text{temporally homogeneous version } (T=\infty) \text{ of noncolliding BM} \\
&= \quad \text{the harmonic transform of the absorbing BM} \\
& \qquad \text{in a Weyl chamber of type } A_{\xi(\mathbb{R})-1} \\
& \qquad \quad \mathbb{W}^{\mathrm{A}}_{\xi(\mathbb{R})} = \{\boldsymbol{x} \in \mathbb{R}^{\xi(\mathbb{R})} : x_1 < x_2 < \cdots < x_{\xi(\mathbb{R})}\} \\
& \qquad \text{with a harmonic function given by} \\
& \qquad \quad h(\boldsymbol{x}) = \prod_{1 \le j < k \le \xi(\mathbb{R})} (x_k - x_j) = \det_{1 \le j,k \le \xi(\mathbb{R})} [x_j^{k-1}].
\end{aligned}
$$

$$
p_N(t, \boldsymbol{y}|\boldsymbol{x}) = \frac{h_N(\boldsymbol{y})}{h_N(\boldsymbol{x})} \det_{1 \le j,k \le N} \Big[p(t, y_j|x_k) \Big], \quad p(t,y|x) = \frac{1}{\sqrt{2\pi t}} e^{-(x-y)^2/2t}.
$$

- $0 < t < \infty, \xi = \sum_{j=1}^{\xi(\mathbb{R})} \delta_{u_j}, \boldsymbol{u} = (u_1, u_2, \dots, u_{\xi(\mathbb{R})}) \in \mathbb{W}^{\mathrm{A}}_{\xi(\mathbb{R})}$

  for g : measurable symmetric function on $\mathbb{R}^{\xi(\mathbb{R})}$

$$\begin{aligned} \mathbb{E}_{\xi}[g(\boldsymbol{X}(t))] &= \mathrm{E}^{(\mathrm{R})}_{\boldsymbol{u}} \left[\mathbf{1}(\tau > t) g(\boldsymbol{V}(t)) \frac{h(\boldsymbol{V}(t))}{h(\boldsymbol{u})} \right] \\ &= \mathrm{E}^{(\mathrm{R})}_{\boldsymbol{u}} \left[\mathbf{1}(\tau > t) g(\boldsymbol{V}(t)) \frac{|h(\boldsymbol{V}(t))|}{h(\boldsymbol{u})} \right], \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{V}(t) &= (V_1(t), V_2(t), \dots, V_{\xi(\mathbb{R})}(t)) \\ & \qquad V_j(t) : \text{ indep. one-dim. standard BM} \\ & \qquad\quad V_j(0) = u_j, \quad 1 \leq j \leq \xi(\mathbb{R}) \\ \mathrm{E}^{(\mathrm{R})}_{\boldsymbol{u}}[\,\cdot\,] &= \text{expectation w.r.t. } \boldsymbol{V}(t) \\ \tau &= \inf\{t > 0 : \boldsymbol{V}(t) \notin \mathbb{W}^{\mathrm{A}}_{\xi(\mathbb{R})}\}. \end{aligned}$$

- By the Karlin-McGregor-LGV formula for the absorbing BM in $\mathbb{W}^{\mathrm{A}}_{\xi(\mathbb{R})}$

$$\begin{aligned}(\mathrm{RHS}) &= \mathrm{E}^{(\mathrm{R})}_{\boldsymbol{u}}\left[\mathbf{1}_{\mathbb{W}^{\mathrm{A}}_{\xi(\mathbb{R})}}(\boldsymbol{V}(t))g(\boldsymbol{V}(t))\frac{|h(\boldsymbol{V}(t))|}{h(\boldsymbol{u})}\right] \\ &= \mathrm{E}^{(\mathrm{R})}_{\boldsymbol{u}}\left[\sum_{\sigma\in S_{\xi(\mathbb{R})}}\mathrm{sgn}(\sigma)g(\boldsymbol{V}(t))\mathbf{1}_{\mathbb{W}^{\mathrm{A}}_{\xi(\mathbb{R})}}(\sigma(\boldsymbol{V}(t)))\frac{|h(\boldsymbol{V}(t)|}{h(\boldsymbol{u})}\right]\end{aligned}$$

$h(\boldsymbol{V}(t))$

$$\sigma(\boldsymbol{u}) \equiv (u_{\sigma(1)}, u_{\sigma(2)}, \dots, u_{\sigma(\xi(\mathbb{R}))}), \sigma \in S_{\xi(\mathbb{R})}$$

- KM-LGV $\oplus$ h-transform

$$\mathbb{E}_{\xi}[g(\boldsymbol{X}(t))] = \mathrm{E}^{(\mathrm{R})}_{\boldsymbol{u}}\left[g(\boldsymbol{V}(t))\frac{h(\boldsymbol{V}(t))}{h(\boldsymbol{u})}\right]$$

singed weight on paths of BMs

4.2 Complex BM representation

Our three observations

[**1**] complexification

$$
\begin{aligned}
V_j(t) \quad &\Longrightarrow \quad Z_j(t) = V_j(t) + iW_j(t), 1 \le j \le \xi(\mathbb{R}) \\
& \qquad W_j(t) : \text{indep. one-dim. standard BM's} \\
& \qquad W_j(0) = 0 \\
\boldsymbol{Z}(t) \quad &= \quad (Z_1(t), Z_2(t), \dots, Z_{\xi(\mathbb{R})}(t)) \\
\mathbf{E}_{\boldsymbol{u}}[\,\cdot\,] \quad &\equiv \quad \mathrm{E}_{\boldsymbol{u}}^{(\mathrm{R})} \otimes \mathrm{E}_{\mathbf{0}}^{(\mathrm{I})}[\,\cdot\,]
\end{aligned}
$$

$$
\mathrm{E}_{\mathbf{0}}^{(\mathrm{I})}[h(\boldsymbol{Z}(t))] = \mathrm{E}_{\mathbf{0}}^{(\mathrm{I})}\left[\det_{1\le j,k\le \xi(\mathbb{R})} (Z_k(t))^{j-1}\right]
$$

by indep. of BMs

$$
\begin{aligned}
&= \det_{1\le j,k\le \xi(\mathbb{R})} \left(\mathrm{E}_{\mathbf{0}}^{(\mathrm{I})}[(Z_k(t))^{j-1}]\right) \\
&= \det_{1\le j,k\le \xi(\mathbb{R})} \left(\mathrm{E}_{\mathbf{0}}^{(\mathrm{I})}[(V_k(t)+iW_k(t))^{j-1}]\right) \\
&= \det_{1\le j,k\le \xi(\mathbb{R})} \left(\underline{\sum_{\ell=0}^{j-1} \binom{j-1}{\ell} V_k(t)^{\ell} i^{j-1-\ell} \mathrm{E}_{\mathbf{0}}^{(\mathrm{I})}[W_k(t)^{j-1-\ell}]}\right)
\end{aligned}
$$

by multi-linearity of det.

$$
= \det_{1\le j,k\le \xi(\mathbb{R})} (V_k(t)^{j-1}) = h(\boldsymbol{V}(t)).
$$

monic polynomial of $V_k(t)$ with degree $j-1$

[2] $\boldsymbol{u} \in \mathbb{W}^{\mathrm{A}}_{\xi(\mathbb{R})}, \boldsymbol{z} \in \mathbb{C}^{\xi(\mathbb{R})}$

$$\frac{h(\boldsymbol{z})}{h(\boldsymbol{u})} = \det_{1 \leq j,k \leq \xi(\mathbb{R})} \left[\prod_{1 \leq \ell \leq \xi(\mathbb{R}): \ell \neq j} \frac{u_\ell - z_k}{u_\ell - u_j} \right]$$

$$= \det_{1 \leq j,k \leq \xi(\mathbb{R})} \left[\prod_{1 \leq \ell \leq \xi(\mathbb{R}): \ell \neq j} \left(1 - \frac{z_k - u_j}{u_\ell - u_j} \right) \right]$$

$$\xi = \sum_{j=1}^{\xi(\mathbb{R})} \delta_{u_j}$$

$$\prod_{x \in \mathrm{supp}\ \xi \cap \{u_j\}^{\mathrm{c}}} \left(1 - \frac{z_k - u_j}{x - u_j} \right) \equiv \Phi_\xi^{u_j}(z_k), \quad z_k \in \mathbb{C}$$

the Weierstrass canonical product with genus 0
(Hadamard theorem)

an entire function with zeros at supp $\xi \cap \{u_j\}^{\mathrm{c}}$
analytic everywhere in $\mathbb{C}$

[3] Map by an entire function gives a conformal transformation.

$$
\begin{aligned}
\Phi_\xi^{u_j}(\cdot) \quad &: \quad \text{a conformal transformation} \\
\Phi_\xi^{u_j}(Z_k(\cdot)) \quad &: \quad 1 \leq j, k \leq \xi(\mathbb{R}) \\
&\quad\;\; \text{indep. conformal local martingales} \\
&\quad\;\; = \text{time changes of complex BMs}
\end{aligned}
$$

Theorem 4.1

Suppose that $\xi(\cdot) = \sum_{j=1}^{\xi(\mathbb{R})} \delta_{u_j}(\cdot) \in \mathfrak{M}$ with $\xi(\mathbb{R}) \in \mathbb{N}$. Let $0 < t < T < \infty$. For any $\mathcal{F}(t)$-measurable function F, we have

$$\mathbb{E}_\xi[F(\Xi(\cdot,\cdot))] = \mathbf{E}_{\boldsymbol{u}}\left[F\left(\sum_{j=1}^{\xi(\mathbb{R})} \delta_{V_j(\cdot)}(\cdot) \right) \det_{1 \leq j,k \leq \xi(\mathbb{R})} \left[\Phi_\xi^{u_j}(Z_k(T)) \right] \right].$$

We call this the complex BM (CBM) representation of the Dyson model (the non-colliding BM).

4.3 Eynard-Mehta-type correlation kernel

- $\chi \in \mathrm{C}_0(\mathbb{R}) \equiv \{\text{cont. real-valued function with compact support}\}$

$$\mathbb{E}_\xi \left[\int_{\mathbb{R}} \chi(x) \Xi(t, dx) \right] \equiv \int_{\mathbb{R}} dx\, \chi(x) \rho_\xi(t, x)$$

By Theorem 4.1

$$\begin{aligned}
& \int_{\mathbb{R}^{\xi(\mathbb{R})}} \xi^{\otimes \xi(\mathbb{R})}(d\boldsymbol{v}) \mathbf{E}_{\boldsymbol{v}} \left[\chi(V_1(t)) \det_{1 \leq j,k \leq \xi(\mathbb{R})} [\Phi_\xi^{u_j}(Z_k(T))] \right] \\
= & \int_{\mathbb{R}} \xi(dv) \mathbf{E}_v [\chi(V(t)) \Phi_\xi^v(Z(t))] \\
= & \int_{\mathbb{R}} \xi(dv) \mathbf{E}_v [\chi(V(t)) \Phi_\xi^v(V(t) + iW(t))] \\
= & \int_{\mathbb{R}} \xi(dv) \int_{\mathbb{R}} dx\, p(t, x|v) \int_{\mathbb{R}} dw\, p(t, w|0) \chi(x) \Phi_\xi^v(x + iw) \\
= & \int_{\mathbb{R}} dx\, \mathcal{G}_{t,t}(x, x),
\end{aligned}$$

for $j \neq 1$

$$\begin{aligned}
\mathbf{E}_{\boldsymbol{v}}[\Phi_\xi^{v_j}(Z_k(T))] &= \mathbf{E}_{\boldsymbol{v}}[\Phi_\xi^{v_j}(Z_k(0))] \\
&= \Phi_\xi^{v_j}(v_k) = \delta_{jk}
\end{aligned}$$

martingale property

where

$$p(t, y|x) = \frac{1}{\sqrt{2\pi t}} e^{-(x-y)^2/2t}$$

$$\mathcal{G}_{s,t}(x, y) \equiv \int_{\mathbb{R}} \xi(dv) p(s, x|v) \int_{\mathbb{R}} dw\, p(t, w|0) \Phi_\xi^v(y + iw)$$

$$\boxed{\rho_\xi(t, x) = \mathcal{G}_{t,t}(x, x) \quad \text{for any initial configuration } \xi \in \mathfrak{M}_0, \xi(\mathbb{R}) \in \mathbb{N}.}$$

- Two-time measurement at $0 < s < t < T < \infty$.
  By Theorem 4.1

$$\mathbb{E}_\xi[g_s(\boldsymbol{X}(s))g_t(\boldsymbol{X}(t))] = \mathbf{E}_{\boldsymbol{u}}\left[g_s(\boldsymbol{V}(s))g_t(\boldsymbol{V}(t)) \det_{1\le j,k\le \xi(\mathbb{R})}(\Phi_\xi^{u_j}(Z_k(T)))\right]$$

symmetric functions

Here we set

$$g_s(\boldsymbol{x}) = \sum_{j=1}^{\xi(\mathbb{R})} \chi_s(x_j), \quad g_t(\boldsymbol{x}) = \sum_{j=1}^{\xi(\mathbb{R})} \chi_t(x_j), \quad \chi_s, \chi_t \in \mathrm{C}_0(\mathbb{R})$$

Then

$$\begin{aligned}
(\text{LHS}) &= \sum_{j=1}^{\xi(\mathbb{R})}\sum_{k=1}^{\xi(\mathbb{R})} \mathbb{E}_\xi[\chi_s(X_j(s))\chi_t(X_k(t))] \\
(\text{RHS}) &= \sum_{j=1}^{\xi(\mathbb{R})}\sum_{k=1}^{\xi(\mathbb{R})} \mathbf{E}_{\boldsymbol{u}}\left[\chi_s(V_j(s))\chi_t(V_k(t)) \det_{1\le p,q\le \xi(\mathbb{R})}(\Phi_\xi^{u_p}(Z_q(T)))\right] \\
&= \sum_{1\le j,k\le\xi(\mathbb{R}):j\ne k} \mathbf{E}_{(u_j,u_k)}\left[\chi_s(V_j(s))\chi_t(V_k(t)) \det\begin{vmatrix} \Phi_\xi^{u_j}(Z_j(T)) & \Phi_\xi^{u_j}(Z_k(T)) \\ \Phi_\xi^{u_k}(Z_j(T)) & \Phi_\xi^{u_k}(Z_k(T)) \end{vmatrix}\right] \\
&\quad + \sum_{j=1}^{\xi(\mathbb{R})} \mathbf{E}_{u_j}\left[\chi_s(V_j(s))\chi_t(V_j(t))\Phi_\xi^{u_j}(Z_j(T))\right] \\
&= \int_{\mathbb{R}^2} \xi^{\otimes 2}(d\boldsymbol{v})\mathbf{E}_{(v_1,v_2)}\left[\chi_s(V_1(s))\chi_t(V_2(t)) \det\begin{vmatrix} \Phi_\xi^{v_1}(Z_1(s)) & \Phi_\xi^{v_1}(Z_2(t)) \\ \Phi_\xi^{v_2}(Z_1(s)) & \Phi_\xi^{v_2}(Z_2(t)) \end{vmatrix}\right] \\
&\quad + \int_{\mathbb{R}} \xi(dv)\mathbf{E}_v\left[\chi_s(V(s))\chi_t(V(t))\Phi_\xi^{v}(Z(t))\right], \\
&\qquad 0 < s < t < T.
\end{aligned}$$

by martingale property

We can see that

$$\begin{aligned}
\text{(RHS)} &= \int_{\mathbb{R}} dx_1 \int_{\mathbb{R}} dx_2 \, \chi_s(x_1)\chi_t(x_2) \det \begin{vmatrix} \mathcal{G}_{s,s}(x,x) & \mathcal{G}_{s,t}(x,y) \\ \mathcal{G}_{t,s}(y,x) & \mathcal{G}_{t,t}(y,y) \end{vmatrix} \\
&\quad + \int_{\mathbb{R}} dx_1 \int_{\mathbb{R}} dx_2 \, \chi_s(x_1)\chi_t(x_2)\mathcal{G}_{s,t}(x,y)p(t-s,y|x) \\
&= \int_{\mathbb{R}} dx_1 \int_{\mathbb{R}} dx_2 \, \chi_s(x_1)\chi_t(x_2) \det \begin{vmatrix} \mathcal{G}_{s,s}(x,x) & \mathcal{G}_{s,t}(x,y) \\ \mathcal{G}_{t,s}(y,x) - p(t-s,y|x) & \mathcal{G}_{t,t}(y,y) \end{vmatrix}
\end{aligned}$$

Corollary 4.2

Let

$$\mathbb{K}_\xi(s,x;t,y) = \mathcal{G}_{s,t}(x,y) - \mathbf{1}(s>t)p(s-t,x|y).$$

Then, for any $M \in \mathbb{N}, 1 \le N_m \le \xi(\mathbb{R}), 1 \le m \le M$,
$0 < t_1 < t_2 < \cdots < t_M < T < \infty$

$$\sum_{\mathbb{J}_m \subset \mathbb{I}_{\xi(\mathbb{R})}, \sharp\mathbb{J}_m = N_m, 1 \le m \le M} \mathbb{E}_\xi \left[\prod_{m=1}^{M} \prod_{j_m \in \mathbb{J}_m} \chi_{t_m}(X_{j_m}(t_m)) \right]$$

$$= \int_{\prod_{m=1}^{M} \mathbb{W}^{\mathrm{A}}_{N_m}} \prod_{m=1}^{M} \left\{ d\boldsymbol{x}^{(m)}_{N_m} \prod_{j=1}^{N_m} \chi_{t_m}(x^{(m)}_j) \right\} \underline{\det_{1 \le j \le N_m, 1 \le k \le N_n, 1 \le m,n \le M} \left[\mathbb{K}_\xi(t_m, x^{(m)}_j; t_n, x^{(n)}_k) \right]}.$$

Here

$$\mathbb{I}_n = \{1,2,\dots,n\}, n \in \mathbb{N}$$
$$\boldsymbol{x}^{(m)}_{N_m} = (x^{(m)}_1, x^{(m)}_2, \dots, x^{(m)}_{N_m}), 1 \le m \le M.$$

multitime correlation function

$$\rho_\xi(t_1, \boldsymbol{x}^{(1)}_{N_1}; \cdots; t_M, \boldsymbol{x}^{(M)}_{N_M}) = \det_{1 \le j \le N_m, 1 \le k \le N_n, 1 \le m,n \le M} \left[\mathbb{K}_\xi(t_m, x^{(m)}_j; t_n, x^{(n)}_k) \right]$$

Eynard-Mehta-type (dynamical) correlation kernel
Eynard-Mehta (1998)
Nagao-Forrester (1998)

correlation kernel

$$\begin{aligned}\mathbb{K}_{\xi}(s,x;t,y) &= \mathcal{G}_{s,t}(x,y) - \mathbf{1}(s>t)p(s-t,x|y) \\ &= \int_{\mathbb{R}} \xi(dv) p(s,x|v) \int_{\mathbb{R}} dw\, p(t,w|0) \Phi_{\xi}^{v}(y+iw) - \mathbf{1}(s>t)p(s-t,x|y)\end{aligned}$$

asymmetric kernel

determinantal point process on $[0,\infty) \times \mathbb{R}$
(on the spatio-temporal plane)

determinantal process

Proof of Corollary 4.2
By Theorem 4.1

$$
\begin{aligned}
\text{(LHS)} &= \sum_{\ell=\max\{N_m\}}^{\xi(\mathbb{R})} \sum_{\mathbb{J}_m \subset \mathbb{I}_\ell, \sharp \mathbb{J}_m = N_n, 1 \leq m \leq M} \mathbf{1}\left(\bigcup_{m=1}^{M} \mathbb{J}_m = \mathbb{I}_\ell \right) \\
&\quad \times \int_{\mathbb{W}_\ell^{\mathrm{A}}} \xi^{\otimes \ell}(d\boldsymbol{v}) \mathbf{E}_{\boldsymbol{v}} \left[\prod_{m=1}^{M} \prod_{j_m \in \mathbb{J}_m} \chi_{t_m}(V_{j_m}(t_m)) \det_{j,k \in \mathbb{I}_\ell} [\Phi_\xi^{v_j}(Z_k(T))] \right] \\
&= \sum_{\mathbb{I}_{N_1} \subset \mathbb{M} \subset \mathbb{I}_{\xi(\mathbb{R})}} \frac{\prod_{m=1}^{M} \sharp(\mathbb{J}_m \cap \mathbb{I}_{N_{m-1}})! \sharp \mathbb{M}!}{\prod_{m=1}^{M} N_m!} \sum_{(\mathbb{J}_m)_{m=1}^{M} \subset \mathcal{J}(\{N_m\}_{m=1}^{M})} \mathbf{1}\left(\bigcup_{m=1}^{M} \mathbb{J}_m = \mathbb{M} \right) \\
&\quad \times \int_{\mathbb{W}_{\sharp \mathbb{M}}^{\mathrm{A}}} \xi^{\otimes \mathbb{M}}(d\boldsymbol{v}) \mathbf{E}_{\boldsymbol{v}} \left[\prod_{m=1}^{M} \prod_{j_m \in \mathbb{J}_m} \chi_{t_m}(V_{j_m}(t_m)) \det_{j,k \in \mathbb{M}} [\Phi_\xi^{v_j}(Z_k(T))] \right] \\
&= \text{(RHS)}.
\end{aligned}
$$

Here

$$
\begin{aligned}
\mathcal{J}(\{N_m\}_{m=1}^{M}) \quad : \quad & \text{a collection of all series of index sets } (\boldsymbol{J}_1, \dots, \boldsymbol{J}_M) \\
\text{s.t.} \quad & \boldsymbol{J}_1 = \mathbb{I}_{N_1}, \boldsymbol{J}_m \subset \mathbb{I}_{\sum_{m=1}^{M} N_m}, 2 \leq m \leq M, \\
& \boldsymbol{J}_m \cap \mathbb{I}_{\sum_{k=1}^{\ell} N_k \setminus \sum_{k=1}^{\ell-1} N_k} \subset \boldsymbol{J}_\ell, 1 \leq \ell < m \leq M, \\
& \sharp \boldsymbol{J}_m = N_m, 1 \leq m \leq M.
\end{aligned}
$$

5^{th} day: Open/Future Problems

In the last day,

I am planning to discuss some

Open/Future Problems,

which are related with

‘noncolliding diffusion processes and random matrices’.

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