Detailed program

1 Introduction by Prof. Zuber (Week 1, 5 hours)

An introduction to Random Matrices

1. General Features,
2. Computational Techniques, (including "angular matrix integrals")
3. Critical limits
4. Complex or normal matrices and applications

2 Course by Prof. Majumdar (Week 1, 10 hours)

Extreme Value Statistics in Brownian Motions and Random Matrices

I will first consider the simple one dimensional Brownian motion and introduce the method of path integrals to study various properties of the Brownian motion: (i) the statistics of first-passage time (ii) the statistics of maximum etc. The path-integral method makes a beautiful connection to a free particle problem in quantum mechanics. Next we will introduce various constraints on the Brownian motion and study the properties of a variety of constrained Brownian motions such as (a) Brownian bridge (b) Brownian excursion (c) Brownian meander etc. We will see how the path-integral method can be very nicely adapted to study these constrained processes.

Next I'll change subject and discuss some basic properties of random matrices, in particular focusing on the extreme properties, i.e., on the distribution of the largest eigenvalue of a random matrix. I'll discuss various recent developements on the subject, in particular, the so called Tracy-Widom distribution.

Next, we will connect up these two apparently different subjects of Brownian motions and random matrices. We will generalize the path-integral method for a single Brownian motion to the case of multiple Brownian motions, with or without interaction. In particular, when the Brownian motions are non-intersecting, we will see how a description in terms of random matrices appear in this problem. Then using the results from the random matrix theory, we will derive some exact asymptotic results for the extreme variables in a system on non-intersecting Brownian motions. We will also see how similar models appear in the Gauge theory in high energy physics. The techniques that we will discuss will thus have very broad applications.

3 Course by Prof. Krattenthaler (Week 1, 10 hours)

THE COMBINATORICS OF NON-INTERSECTING LATTICE PATHS

Non-intersecting lattice paths in mathematics and physics have an interesting (and involved) history: in their most general form, they appear for the first time (under the name "pairwise node disjoint paths") in 1973 in a paper on representations of matroids by Bernt Lindström. In the 1980s, three different groups "rediscovered" non-intersecting lattice paths independently, guided by concrete applications that they had in mind: Ira Gessel and Xavier Viennot (and followers) used non-intersecting lattice paths in order to enumerate plane partitions and various kinds of tableaux, a group of combinatorial mathematicians and chemists (John, Sachs, Gronau, Just, Schade, Scheffler, and Wojciechowski) used them in order to analyse Pauling's bond order in benzenoid hydrocarbon molecules, and Michael Fisher introduced them under the name of "vicious walkers" into Statistical Mechanics in order to model wetting and melting, thereby finding as well many followers. In fact, continuous versions had already been looked at by Karlin and McGregor around 1960 (in probability), and by J. C. Slater around 1930 and by de Gennes in 1968 (in physics). Confusing as all this may appear, there is at least now a clear picture of what the fundamental results concerning the enumeration of non-intersecting lattice paths are. Moreover, the above historical sketch underlines nicely that these objects are of importance and interest in several areas of Combinatorics, Probability, and Statistical Physics.

In the first part of these lectures, I shall introduce non-intersecting lattice paths, and I shall present the basic results: the Lindström-Gessel-Viennot theorem (as I like to call it), the Okada-Stembridge theorems and the minor summation formula of Ishikawa-Wakayama, plus some variations. This part will also include a discussion of the closely related Gessel-Zeilberger theorem of walks in Weyl chambers, including the necessary background on reflection groups.

In the second part, I shall discuss applications: these concern mainly the afore-mentioned plane partitions and tableaux, and rhombus tilings. When discussing tableaux, I also hope to touch upon topics such as symmetric functions and, in particular, Schur functions and other classical group characters.

The final part will be concerned with the asymptotic properties of a fixed number of non-intersecting lattice paths as the length of the paths tends to infinity.

4 Course by Prof. Katori, Week 2, 10 hours

SYMMETRIES AND STRUCTURES OF MATRIX-VALUED STOCHASTIC PROCESSES AND NONCOLLIDING DIFFUSION PROCESSES

In my lectures, first I give a proof of the equivalence in distribution of eigenvalue processes of Hermitian matrix-valued stochastic processes and one-dimensional diffusion processes of particles conditioned never to collide with each other, and then I show that the consequences of this equivalence are very rich both in mathematics and physics.

The following correspondence between the random matrix (RM) theory and the present study of stochastic processes is explained : (i) Addition of higher symmetries to RM ensembles corresponds to imposing spatial boundary conditions to stochastic processes, (ii) Two-matrix models corresponds to temporally inhomogeneous versions of processes, (iii) Breaking symmetry by introducing external sources in RM models corresponds to generalizing initial states in processes.

Using the above correspondence, I discuss the 10 classes of RM theory of Altland and Zirnbauer and a variety of Itzykson-Zuber-Harish-Chandra formulas of integrals over unitary groups with additional symmetries. The determinantal and pfaffian structures of spatial and temporal correlations of the processes are derived. In particular, the Eynard-Mehta-type dynamical correlation-kernels, first calculated for two-matrix models, are obtained as direct consequences of the equivalence between eigenvalue processes and noncolliding processes. Theories of multiple orthogonal polynomials and entire functions on the complex plane are introduced to perform infinite-particle limits and the determinantal processes with sine-, Airy-, Bessel-, and Pearcey-kernels are discussed. Related new topics, *e.g.*, the extreme-value distributions studied by Schehr *et. al.* and interlacing phenomena in Dyson's Brownian motion models will be included. My talk is mainly based on the joint work with H. Tanemura.

5 Course by Prof. Johansson (Week 2, 10 hours)

Determinantal processes, random growth and random tilings

I will give an introduction to determinantal point processes and the basic limit processes that come out of discrete or continuous non-colliding paths. These limit processes include the Sine, Airy, Pearcey and Tacnode kernel processes and their extended versions. These processes occur as natural scaling limits in random matrix theory but also in several discrete statistical mechanical models, e.g. random tilings and 1+1-dimensional random growth. I will discuss some of these applications. In some cases these models give rise directly to a determinantal point process by some combinatorial argument, whereas in other cases the determinantal limit processes only occur, or are expected to occur, in the limit.