集中講義
九州大学大学院工学府 エネルギー量子工学専攻
「応用物理学特別講義 II」
2006年6月5日 (月) －7日 (水)

『相転移と自己組織化臨界現象』
講義ノート §1.1

か とり　まこと

香取 眞理

(中央大学理工学部物理学科)

研究分野　　統計物理学

数理物理学

確率論

数理生物学

経済物理学

講義の計画

6月5日（月）

10：30－12：00：自己組織化臨界現象とは　（パワーポイント）

（以下講義は黒板で行います。）

13：00－14：30：アーベル的砂山崩しモデル（ASM）の定義

6月6日（火）

10：30－12：00：再帰的配置と定常分布

13：00－14：30：再帰的配置の総数

15：00－16：30：講演会「量子ウォークの特異な拡散現象」

（パワーポイント）

6月7日（水）

10：30－12：00：禁止部分配置と許容配置

なだれの伝播関数

13：00－14：30：2次元正方格子上のASM

§1.1　森林生態系とイジング模型

Neotropical forest
in Baro Colorado Island, Panama

50 ヘクタール (500 m × 1000 m) の熱帯季節林
Deta: S.P. Hubbell and R. B. Foster, 1986 年発表

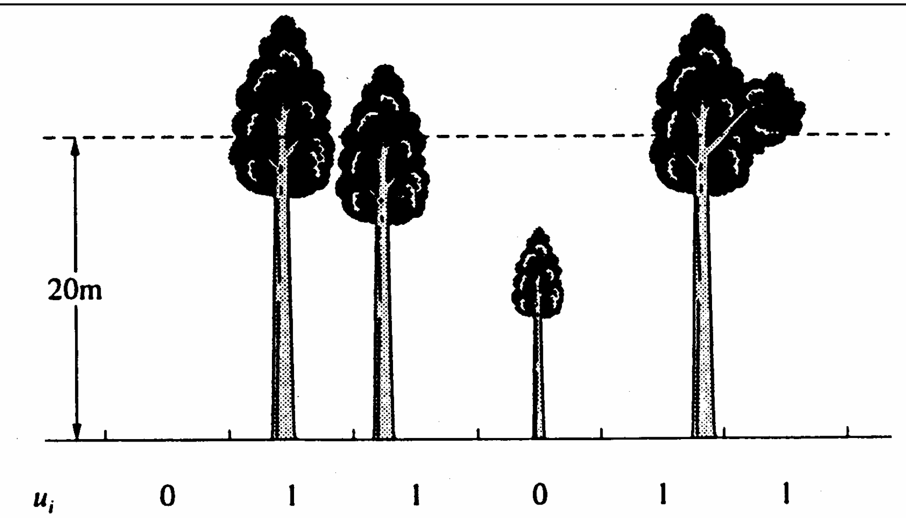


図 2・13　林冠サイトと林冠ギャップサイト

小調査区 (5 m × 5 m) に分割

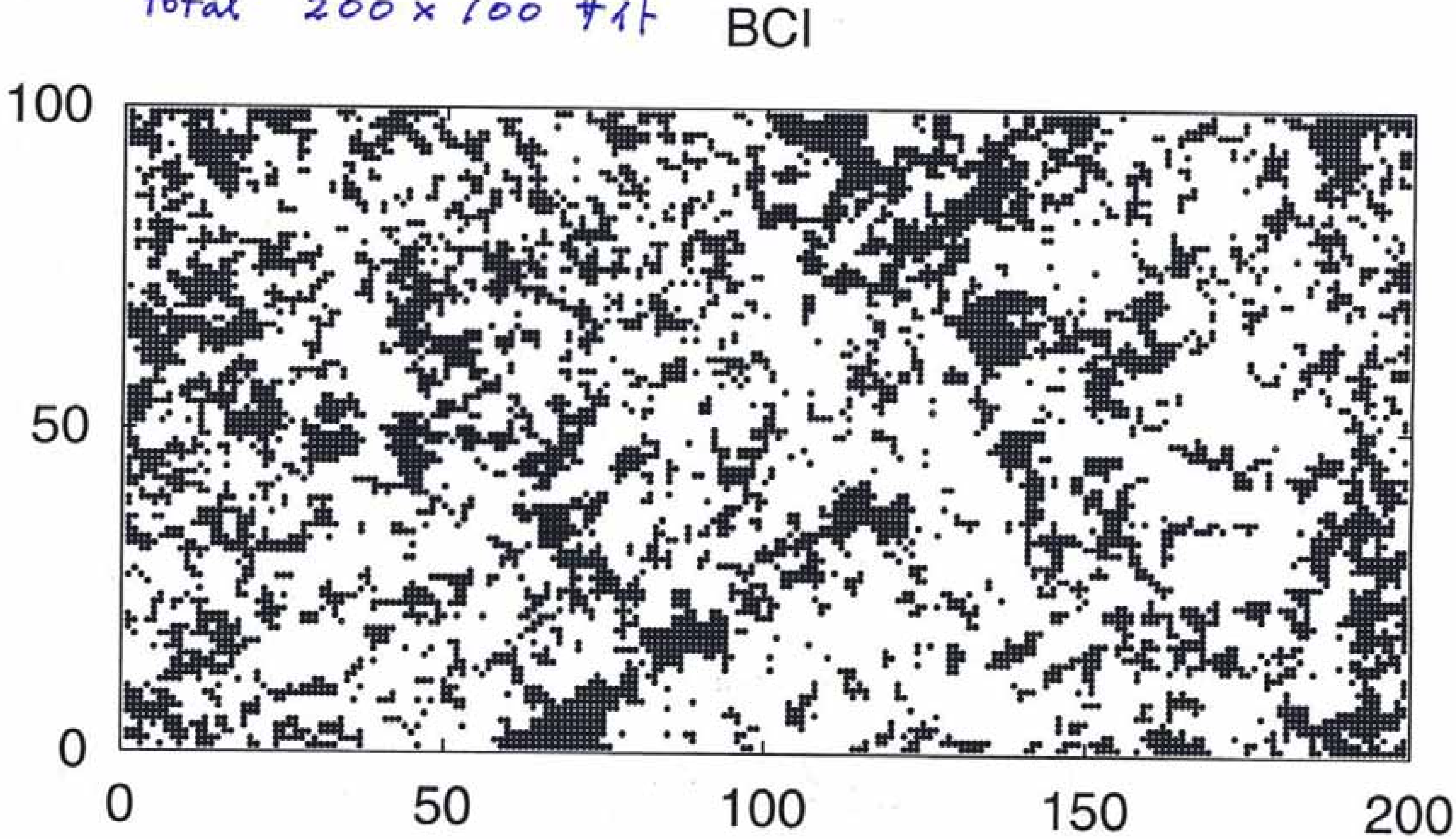
Canopy-gap (林冠ギャップ) サイト　＝　最高樹高　＜ 20 m の小調査区のこと

1983年調査結果

● 林冠ギャップサイト 樹高 < 20m

白 林冠サイト 樹高 > 20m

total 200 × 100 サイト



500m ↓ 5m × 100

1000m → 5m × 200

小川森林保護区 (日本)

555 m × 455 m
小調査区 5 m × 5 m

航空写真からデータ化
5年毎に調査
1976年
1981年
1986年
1991年
田中氏、中静氏
京都大学、森林総研

林冠ギャップサイト
＝最高樹高 < 15 m

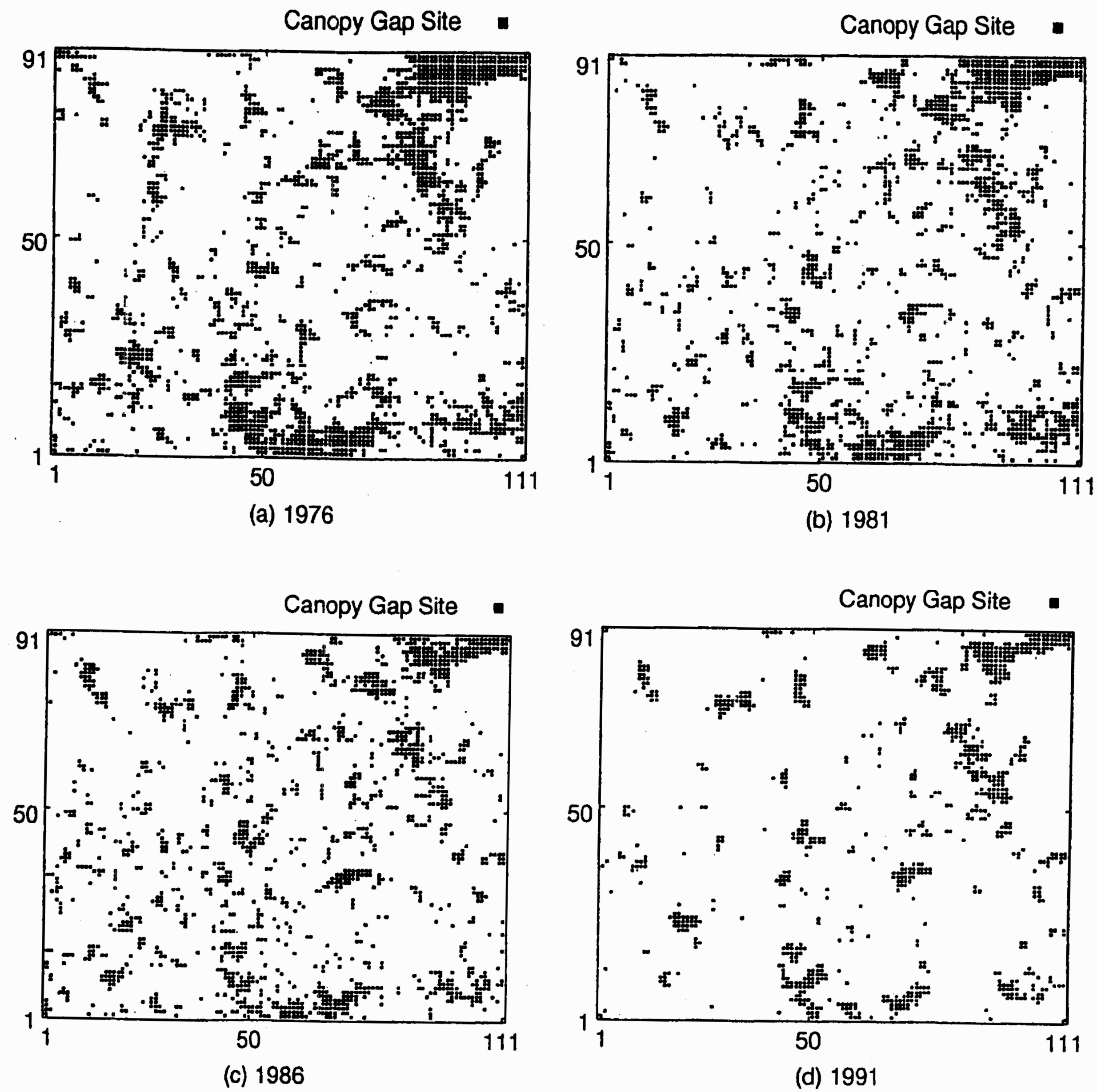


Fig. 3. 555×455 m digitized maps of the deciduous forest on Ogawa Forest Reserve (OFR), Japan, in (a) 1976, in (b) 1981, in (c) 198 and in (d) 1991.[1] Gap sites, 5×5 m subplots in which there is no canopy higher than 15 m, are plotted by black dots.

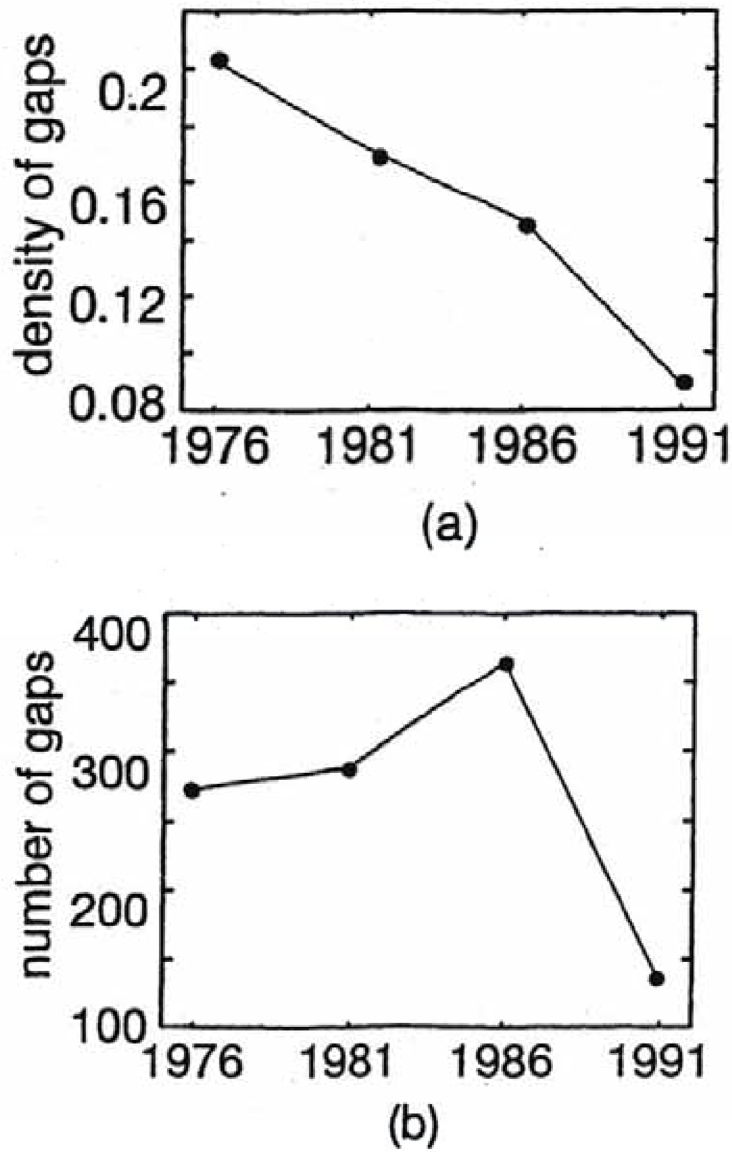


Fig. 5. The time dependence of (a) a density of gaps ρ_0 and (b) a total number of gaps n_0 of OFR.

Barro Colorado Island (1983)

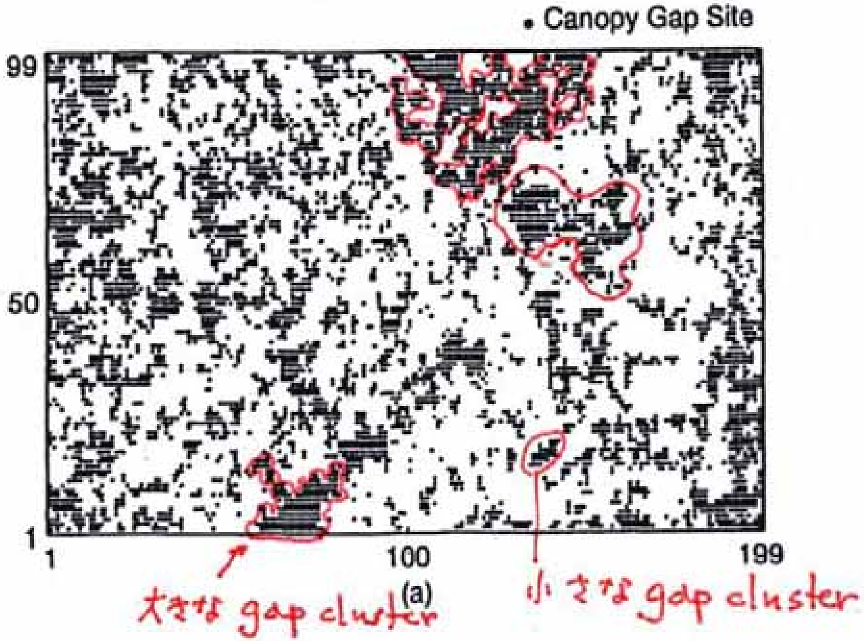


Fig. 1. 1000×500 m digitized map of the neotropical forest in Barro Colorado Island, Panama, in 1983.[2,4] Gap sites, 5×5 m subplots in which vegetation height is less than 20 m, are plotted by black dots.

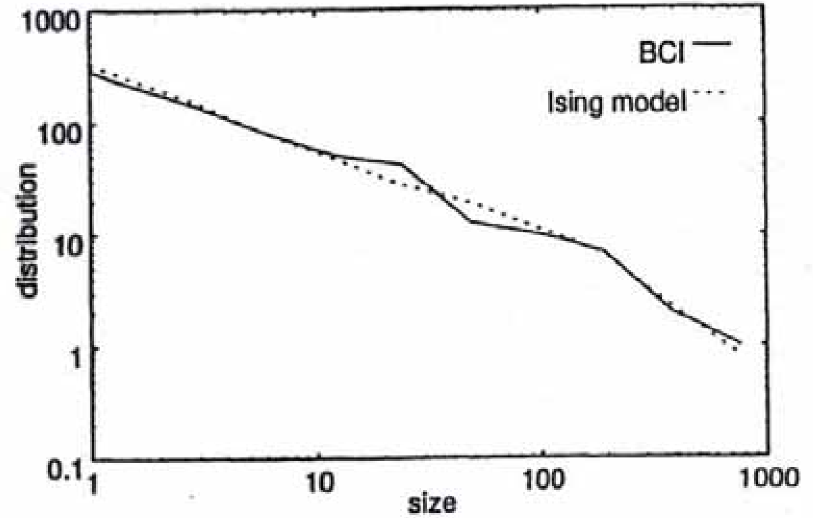


Fig. 2. Log-log plot of the gap-size distribution for BCI and the cluster-size distribution of down spins in the Ising-Gibbs state with $K = 0.37$ and $h = 0.016$.[11] The size of the gap cluster (down-spin cluster) is determined with Neumann neighborhood. For the Ising-Gibbs state, we performed the Monte Carlo simulations on a 700×700 square lattice with the periodic boundary condition and averaged over 10 data, each of which is obtained after discarding 1500 Monte Carlo steps. Clusters contained in a 199×99 region on the lattice are counted and the points in the distribution have been logarithmically binned in boxes of powers

林冠ギャップのかたまり (クラスター、cluster) のサイズ S の分布

両対数グラフでほぼ一直線

Ogawa Forest Reserve
(小川森林保護区)

林冠ギャップの
クラスターサイズ分布.
⇓
どれも
両対数グラフで
ほぼ直線.

分布関数
$f(s) \sim s^{-\tau}$
$(s \leq s_{cut})$

べき乗分布

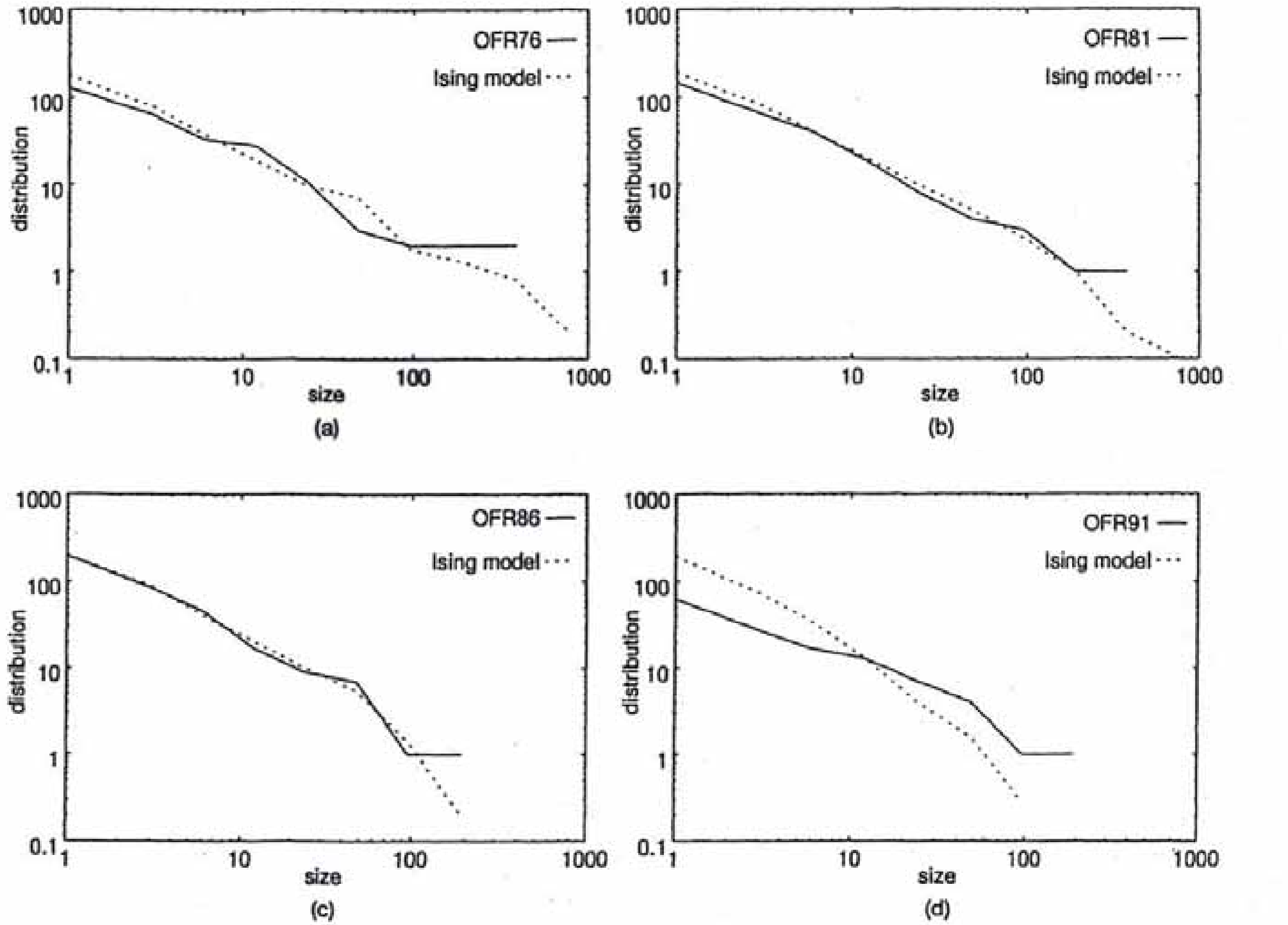


Fig. 4. Log-log plots of the gap-size distributions for OFR in (a) 1976, in (b) 1981, in (c) 1986 and in (d) 1991, and those of the cluster-size distributions of down spins in the Ising-Gibbs states with K and h listed in Table I. For the Ising-Gibbs states, the Monte Carlo simulations were performed and clusters contained in a 111×91 region on the 700×700 lattice were counted. The points in the distribution have been logarithmically binned in boxes of powers of two.

林冠ギャップ
クラスターサイズ分布

$f(s) \sim s^{-\tau}$
$(s \lesssim s_{cut})$

指数 τ
BCI
OFR 86, 76
ともに $\tau \fallingdotseq 1.7$

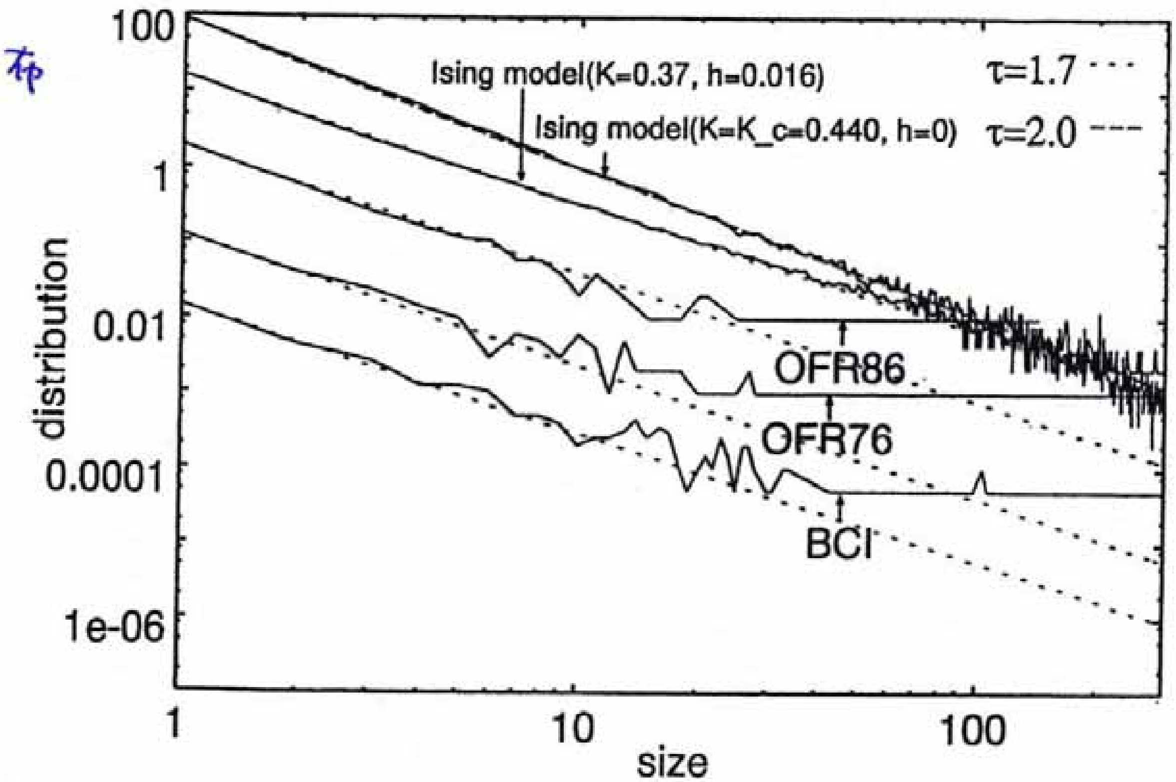
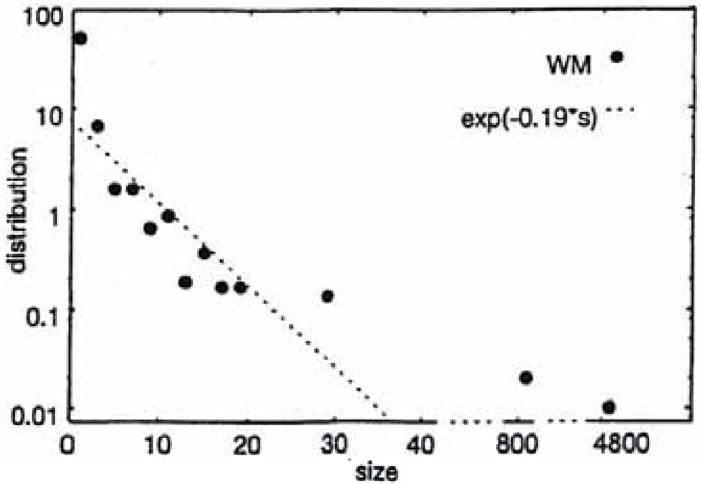


Fig. 6. From the bottom to the top, the gap-size distribution $f_f(s)$ for BCI, $f_f(s)$ for OFR in 1976, in 1986, the cluster-size distribution of down spins $f_{IG}(s)$ of the Ising model with $K = 0.36$ and $h = 0.016$ and $f_{IG}(s)$ with $K = K_c = 0.440$ and $h = 0$. These are shifted upward in order to see them easily. The points in the distributions are not logarithmically binned. The distribution at the top is fitted by a size to the power of 2.0. Other four distributions are fitted by a size to the power of 1.7.

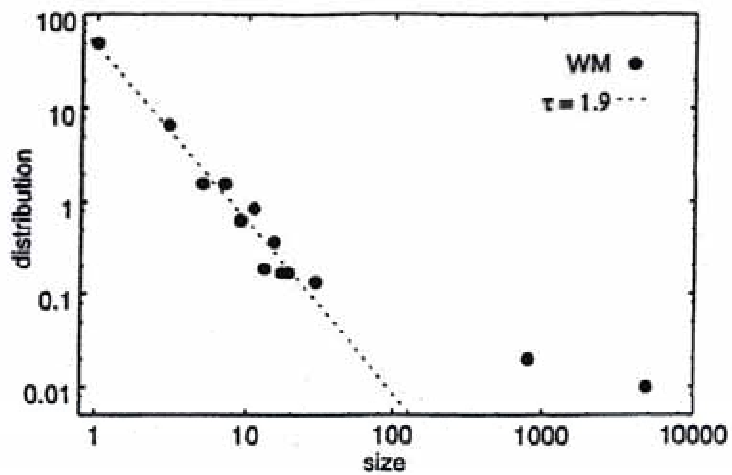


(a)

別の森林データ

Foster-Reiners (1986)

White Mountains, New Hampshire, USA

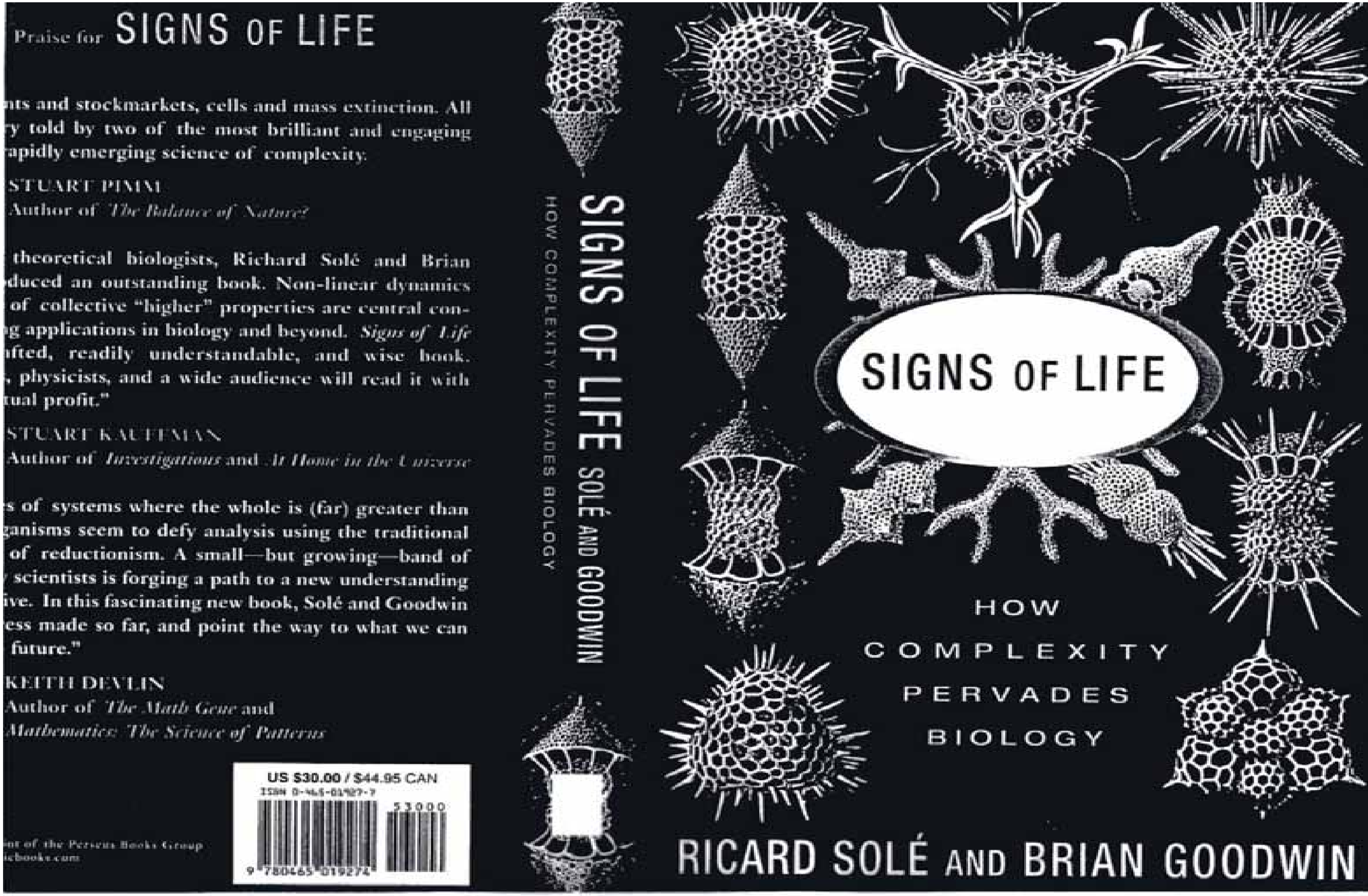


(b)

$f(s) \sim s^{-\tau}$

$\tau \doteqdot 1.9$

Fig. 7. The gap-size distribution $f_f(s)$ for White Mountains (WM), New Hampshire, USA from ref. 3. (In the ordinate of Fig. 1 of ref. 3, 15 should be read as 50.) (a) The semilog plot of $f_f(s)$ for WM. (b) The log-log plot of $f_f(s)$ for WM. The dashed line in (a) denotes the fitted exponential function of a size and that in (b) denotes the fitted algebraic function of a size in the region where sizes are less than 30.



Rechard Sole’らの2000年発行の本

（サンタフェ　複雑系研究所、バルセロナ大）

An Ising Model for the Rainforest

In a recent paper, Makoto Katori and coworkers have shown that the forest canopy dynamics of the Barro Colorado plot can, in fact, be regarded as the result of an Ising-like model[21] (see Chapter 2). Using a simple set of rules directly inspired by field data analysis, Katori et al. reproduced some of the most interesting properties of the Barro Colorado 50 hectare plot. In Figure 7.17 the basic rules are described. Here two types of states are allowed (as with the Ising model): nongap points (here white squares) and gap points (black squares, corresponding to canopy below 20 meters high).

Field data provided the estimation of the transition rates (indicated in the figure below the arrows for each of the six possible cases). Here d is the spontaneous creation of a canopy gap (here $d \approx 0.024$), and δ_k

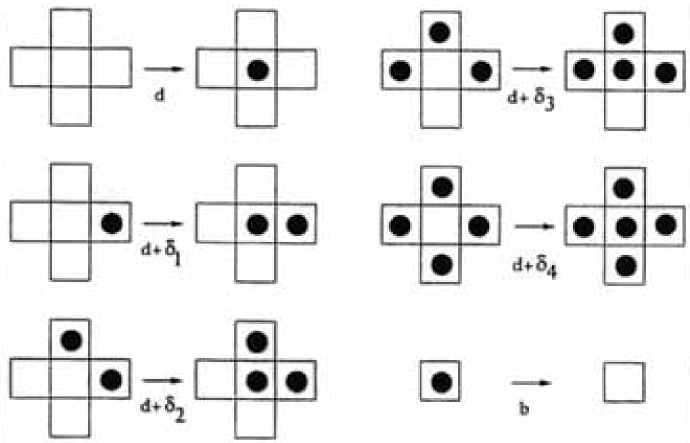


Figure 7.17 Basic rules defined in the Katori's et al. model. Gaps sites are denoted by black circles.

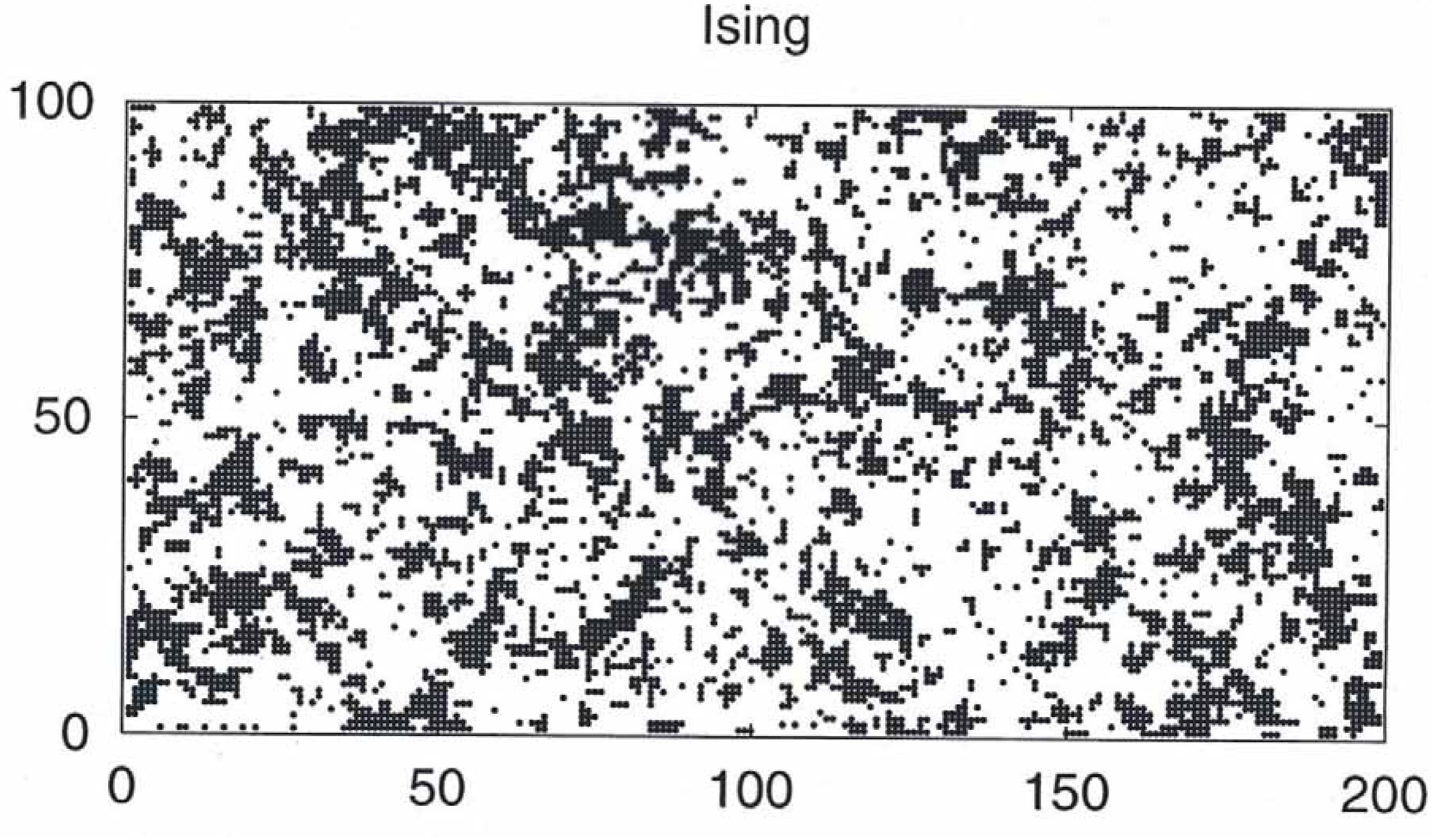
林冠ギャップ生成.消滅の確率モデル
=
Ising模型

indicates the risk of falling trees due to the presence of neighboring gaps. Katori et al. use the simple (and sensible) approximation $\delta_k = k\delta$, where $k = 1, \ldots, 4$ and $\delta = 0.276$. This choice is based in the observation that the presence of nearest canopy gaps strongly increases the fall of neighboring trees either by direct physical effects or because of the strong modifications of local microclimate. The model is completed by introducing a transition rate from a canopy gap point to a noncanopy point due to tree growth ($b = 0.177$).

The dynamics of this model give quite good results. But Katori et al. go a step further and show that in fact, this model is equivalent to an Ising model close to criticality. They estimated the appropiate temperature for the Ising model configuration consistent with Barro Colorado data. An example of the similarity between the Ising model and its rainforest counterpart is shown in Figure 7.15b, to be compared with Figure 7.15a (from Katori et al., 1998). The quantitative agreement between both plots can be shown by means of fractal measures or by plotting the size distribution of canopy gaps. The later is shown in Figure 7.16, where both field data and simulation are shown. These results give strong support to the early conjecture that rainforest dynamics take place close to critical states.[22]

Ising 模型でシミュレートしたBCIの林冠ギャップ分布

$$K = \frac{J}{kT} = 0.37, \quad h = \frac{\mu_B H}{kT} = 0.016$$



磁石は Tc(キュリー温度)を超えると磁力を失う。 12
鉄の場合 770°C くらい。

強磁性体の相転移現象

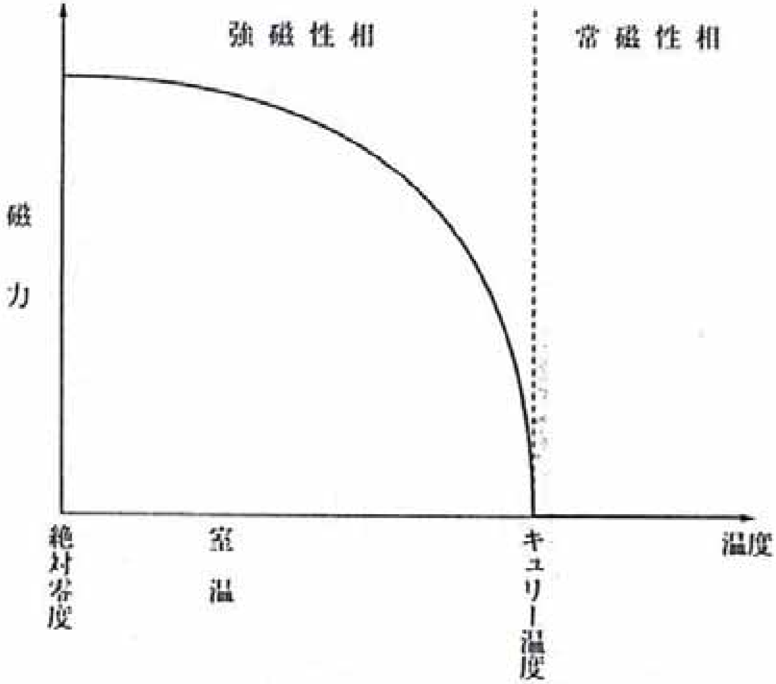


図2・1 磁石の磁力の温度による変化

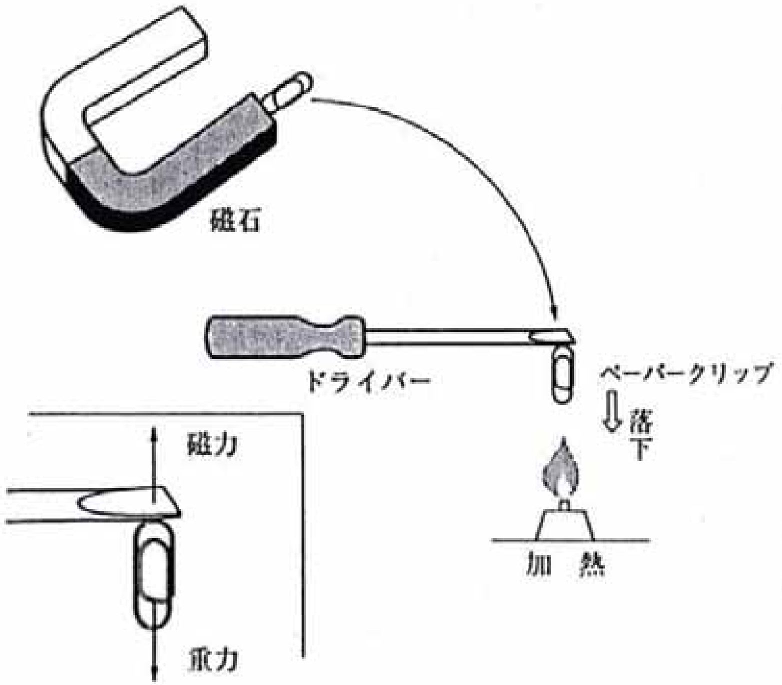


図2・2 簡単な実験

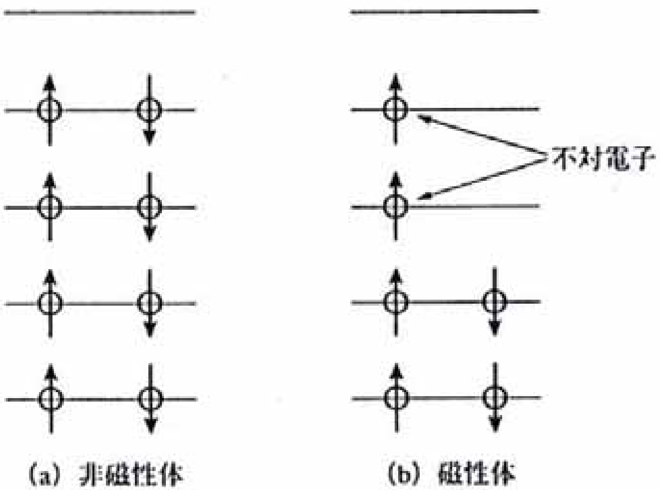


図２・４　磁性体と非磁性体

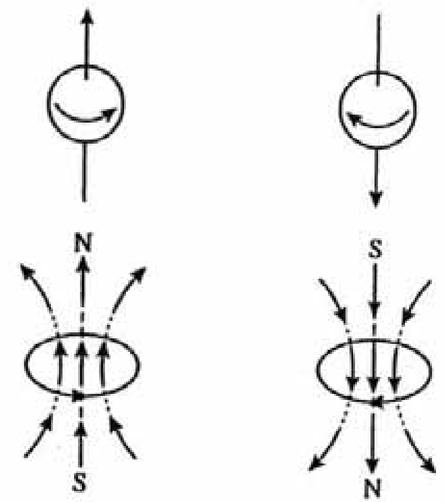


図２・３　電子のスピンの古典的イメージ

非磁性体 … 磁石につかないもの
例）アルミニウム

磁性体 … 磁石につくもの
例）鉄，ニッケル

電荷 $(-1.6\times10^{-19}C)$ を持つ電子は自転（spin）している．

⇓

微小磁力を持つ

$\mu_B = 9.27\times10^{-24}\ J/T$

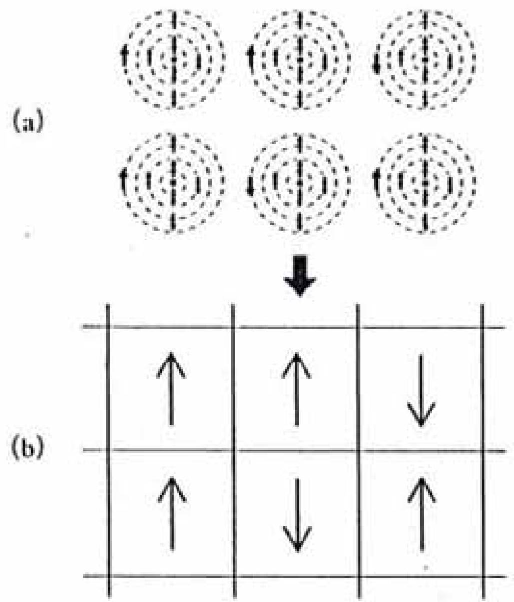


図2・5　粗視化して格子スピン・モデルにする

パウリの排他律

電子などのフェルミ粒子は、同じ状態に

2つの以上の粒子が同時になることは不可能

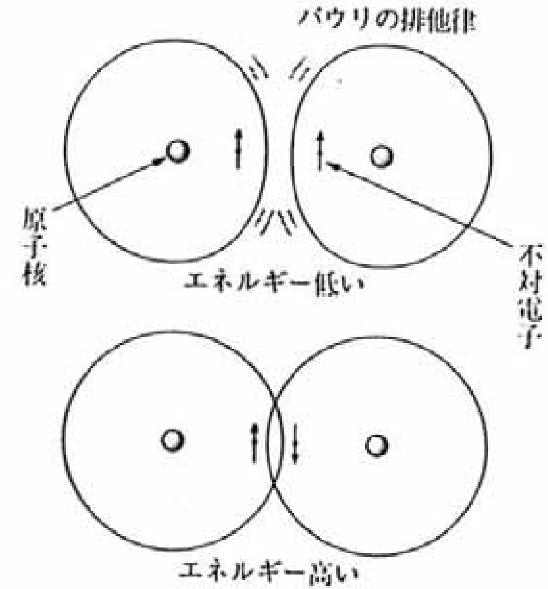


図2・6　パウリの排他律と交換相互作用

同じ状態

= スピンの向きがそろった状態

自転

‖

N−Sの向き。

☆ 近づくと クーロン反発力のため

電気ポテンシャル・エネルギーが高くなる。

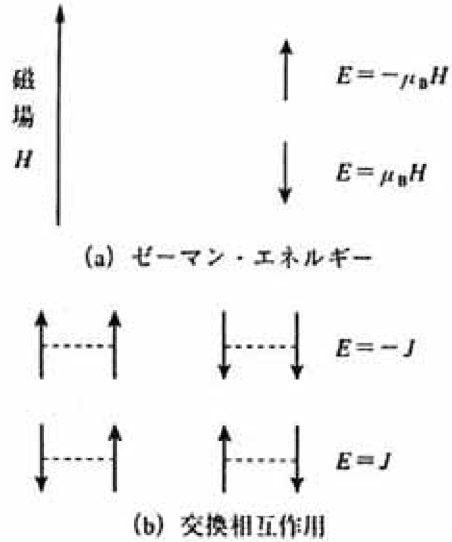


図2・7　スピンの向きとエネルギー

エネルギー

$$E = -J\sum_{i,j} S_i S_j - \mu_B H \sum S_i$$

S_i = サイト i の電子のスピンの向き

$$= \begin{cases} 1 & \cdots\cdots \ \text{N-S} \ \uparrow \\ -1 & \cdots\cdots \ \text{S-N} \ \downarrow \end{cases}$$

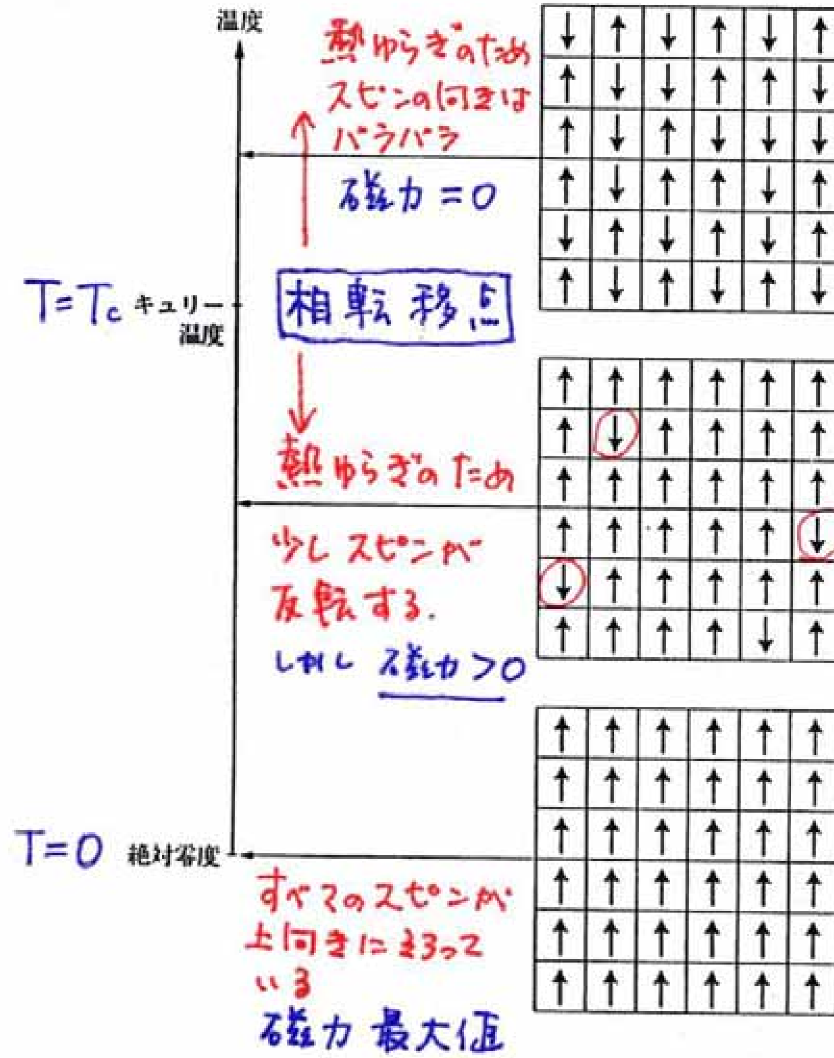
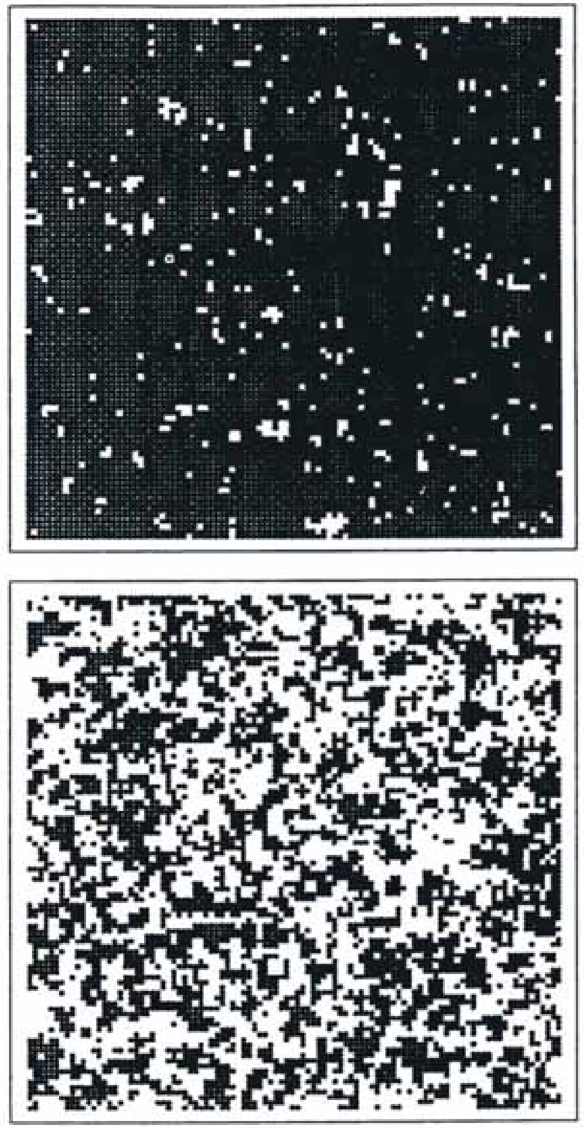


図2・8　スピン配置と温度

(a) $T<T_C$

(b) $T>T_C$



$T=T_c$
Curie温度
‖
相転移温度
‖
臨界点

図 2・10　イジング・モデルのシミュレーション結果

熱力学 (統計力学) の大法則 E ＝エネルギー、T ＝絶対温度、S ＝エントロピーとしたとき

「自由エネルギー最小の原理」 $F = E - T S$

Table 1: List of the parameters K and h which are chosen to satisfy the correlation equalities for BCI in 1983 and for OFR in 1976, 1981, 1986 and 1991.

Data	K	h
BCI(1983)	0.350	0.043
OFR(1976)	0.414	0.052
OFR(1981)	0.428	0.039
OFR(1986)	0.410	0.058
OFR(1991)	0.559	-0.023

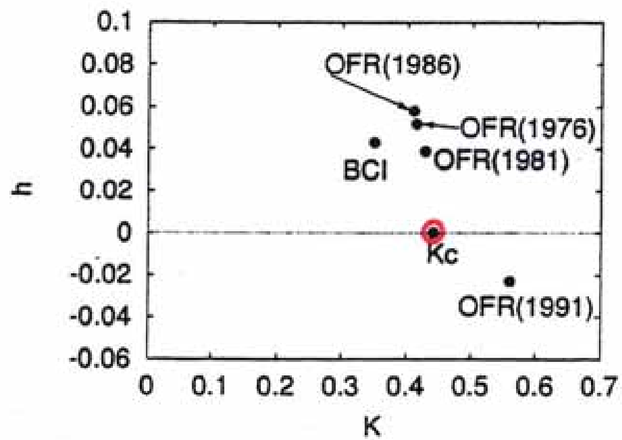


Fig. 4: Plots of the results in Table 1 on a parameter plane (K, h). All plots are located near the critical point $(K_c, 0)$.

2次元正方格子上の Ising 模型.

臨界点 $T_c = 2.269\ J/k$, $H = 0$

$$K = \frac{J}{kT} \Rightarrow K_c = \frac{J}{kT_c} = \frac{1}{2}\log(1+\sqrt{2}) = 0.4406\ldots$$

$$h = \frac{\mu_0 H}{kT} \Rightarrow h_c = 0$$

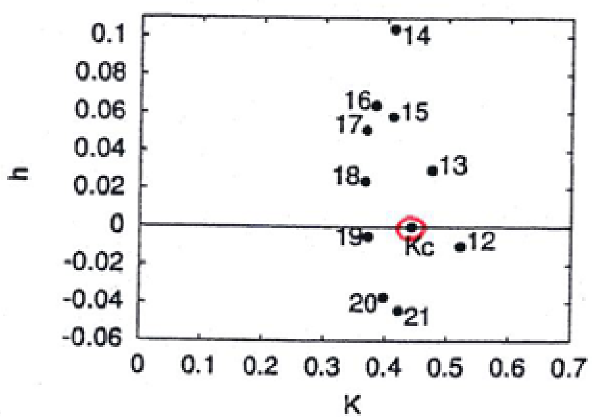
林冠ギャップの定義 = 最高樹高 < 15m

OFRの場合

これを12m〜21mといろいろ変えて比較.

Table 2: List of the parameters K and h for OFR in 1986 ,which are chosen to satisfy the correlation equalities on the configuration of canopy-gap sites for each value of threshold.

Threshold [m]	K	h
12	0.521	-0.010
13	0.474	0.030
14	0.411	0.104
15	0.410	0.058
16	0.382	0.064
17	0.367	0.051
18	0.365	0.024
19	0.371	-0.005
20	0.397	-0.037
21	0.421	-0.044



定義の詳細には依らず(Kc,0)の臨界点の近傍にある.

Fig. 5: Plots of the results in Table 2 on a parameter plane (K,h). The numbers associated with dots indicate the values of threshold height [m]. All plots surround a critical point (K_c,0).

新たな疑問

なぜ森林は相転移点に近い状態を維持しているのか？

↓

自己組織化臨界現象

Self-Organized Criticality (SOC)

の一例

参考文献

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