回転行列のウィグナー公式と多成分量子ウォーク

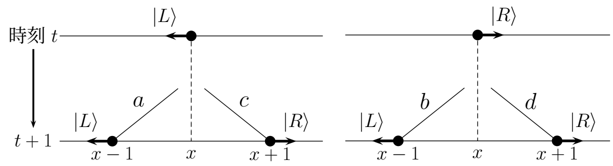
宮崎 玄洋, 香取 眞理, 今野 紀雄[A]
中大理工, 横浜国大工[A]

日本物理学会 2007 年春季大会
2007 年 3 月 20 日
鹿児島大学 (郡元キャンパス, 鹿児島県鹿児島市)

目次

§1 量子ウォークの定義

標準模型 (2 状態, 最近接ホッピング)



■ 量子ウォークの定義式

$$\Psi(x,t+1)=\begin{pmatrix} a\,\Psi_{1/2}(x+1,t)+b\,\Psi_{-1/2}(x+1,t) \\ c\,\Psi_{1/2}(x-1,t)+d\,\Psi_{-1/2}(x-1,t)\end{pmatrix}$$

ただし, $A=\begin{pmatrix} a & b \\ c & d\end{pmatrix}\in \mathrm{SU}(2), \quad \Psi(x,t)=\begin{pmatrix}\Psi_{1/2}(x,t)\\ \Psi_{-1/2}(x,t)\end{pmatrix}$

■ フーリエ変換

$$\Psi(x,t)=\int_{-\pi}^{\pi}\frac{dk}{2\pi}\hat{\Psi}(k,t)e^{\mathrm{i}kx}$$
$$\hat{\Psi}(k,t)=\sum_{x\in\mathbf{Z}}\Psi(x,t)e^{-\mathrm{i}kx}$$

■ t ステップでの波動関数 $(t=0,1,2,\cdots)$

$$\begin{pmatrix}\Psi_{1/2}(x,t)\\ \Psi_{-1/2}(x,t)\end{pmatrix}=\int_{-\pi}^{\pi}\frac{dk}{2\pi}\left[\begin{pmatrix} e^{\mathrm{i}k} & 0 \\ 0 & e^{-\mathrm{i}k}\end{pmatrix}\begin{pmatrix} c\,e^{-\mathrm{i}(\alpha+\gamma)/2} & -s\,e^{-\mathrm{i}(\alpha-\gamma)/2} \\ s\,e^{\mathrm{i}(\alpha-\gamma)/2} & c\,e^{\mathrm{i}(\alpha+\gamma)/2}\end{pmatrix}\right]^t\begin{pmatrix} q_{1/2} \\ q_{-1/2}\end{pmatrix}$$

$$\mathrm{SU}(2)=\left\{A=\begin{pmatrix} a & b\\ c & d\end{pmatrix}=\begin{pmatrix} a & b \\ -b^* & a^*\end{pmatrix}; a,b\in\mathbf{C}, |a|^2+|b|^2=1\right\}$$
$$=\left\{A=\begin{pmatrix} c\,e^{-\mathrm{i}(\alpha+\gamma)/2} & -s\,e^{-\mathrm{i}(\alpha-\gamma)/2} \\ s\,e^{\mathrm{i}(\alpha-\gamma)/2} & c\,e^{\mathrm{i}(\alpha+\gamma)/2}\end{pmatrix}; \begin{array}{l} c=\cos(\beta/2),\ s=\sin(\beta/2) \\ \alpha,\gamma\in[-\pi,\pi),\ \beta\in[-\pi/2,\pi/2)\end{array}\right\}$$

$$\Psi(x,0)=\delta_{x\,0}\begin{pmatrix} q_{1/2}\\ q_{-1/2}\end{pmatrix},\quad |q_{1/2}|^2+|q_{-1/2}|^2=1$$

§2 今野の弱収束の定理

しかし $\cdots$,
ユニタリ行列の固有値の絶対値は 1 $(|\lambda|=1)$ なので
波動関数 $\Psi(x,t)$ 及び確率密度 $P(x,t)$ は $t\to\infty$ の
長時間極限をとっても収束しない

今野の弱収束の定理 (今野紀雄氏 横浜国大工学研究院)

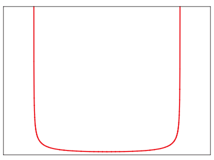
N.Konno: *Quantum Inf. Process* **1**, 345 (2002)
J. Math. Soc. Jpn. **57**, 1179 (2005)

量子ウォークの 擬速度 $\dfrac{X_t}{t}$ の任意のモーメントは
$t\to\infty$ の長時間極限において収束する (弱収束)

今野の分布関数

任意の $r=0,1,2,\cdots$ に対して

$$\lim_{t\to\infty}\left\langle\left(\frac{X_t}{t}\right)^r\right\rangle=\int_{-\infty}^{\infty}dy\,y^r\nu(y)$$



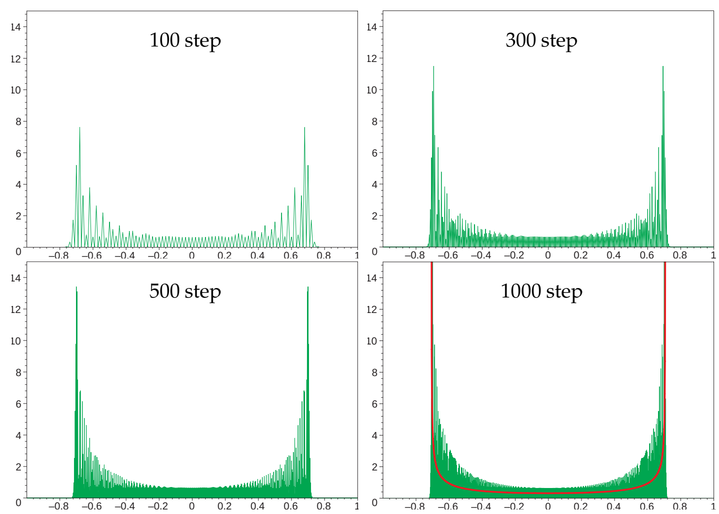
ただしここで, $\nu(y)$ は

$$\mu(x;a)=\frac{\sqrt{1-a^2}}{\pi(1-x^2)\sqrt{a^2-x^2}}\mathbf{1}_{\{|x|<|a|\}}$$

$$\mathcal{M}(x)=1+\left[-\left\{|q_{1/2}|^2-|q_{-1/2}|^2\right\}+2\tau\,\mathrm{Re}\left\{q_{1/2}\overline{q}_{-1/2}e^{-\mathrm{i}\gamma}\right\}\right]x$$

を用いて

$$\nu(y)=\mu\left(y;\cos\tfrac{\beta}{2}\right)\mathcal{M}(y)$$



§3 模型の拡張

量子力学では, 回転は演算子の形で

$$\hat{R}(\alpha,\beta,\gamma) = e^{-\mathrm{i}\,\alpha\hat{J}_3} e^{-\mathrm{i}\,\beta\hat{J}_2} e^{-\mathrm{i}\,\gamma\hat{J}_3}$$

$\hat{\boldsymbol{J}} = (\hat{J}_1, \hat{J}_2, \hat{J}_3)$ は角運動量演算子ベクトル

$$[\hat{J}_k, \hat{J}_\ell] = \mathrm{i}\sum_{m=1}^{3} \varepsilon_{k\ell m}\hat{J}_m$$

$\varepsilon_{k\ell m}$: 完全反対称テンソル, $\quad k,\ell = 1,2,3$

ケットベクトル $|j,m\rangle$, $\quad j = 0, {}^1\!/_2, 1, {}^3\!/_2, \cdots, \quad m = -j, -j+1, \cdots, j$ は

$$\hat{\boldsymbol{J}}^2|j,m\rangle = j(j+1)|j,m\rangle, \quad \hat{J}_3|j,m\rangle = m|j,m\rangle$$

を満たすように定めた $\hat{J}^2 = \sum_{k=1}^{3}\hat{J}_k^2$ と $\hat{J}_3$ の固有状態を意味するものとする. (以下では $\hbar = 1$ としている.)

半整数 j の値を固定して $m, m' = -j, -j+1, \cdots, j$ に対して

$$R^{(j)}_{mm'}(\alpha,\beta,\gamma) = \langle j,m|\hat{R}(\alpha,\beta,\gamma)|j,m'\rangle$$

とする. これを計算すると

$$r^{(j)}_{mm'}(\beta) = \sum_\ell \Gamma(j,m,m',\ell)\left(\cos\frac{\beta}{2}\right)^{2j+m-m'-2\ell}\left(\sin\frac{\beta}{2}\right)^{2\ell+m'-m}$$

$$\Gamma(j,m,m',\ell) = (-1)^\ell \frac{\sqrt{(j+m)!\,(j-m)!\,(j+m')!\,(j-m')!}}{(j-m'-\ell)!\,(j+m-\ell)!\,\ell!\,(\ell+m'-m)!}$$

として

$$R^{(j)}_{mm'}(\alpha,\beta,\gamma) = e^{-\mathrm{i}\,\alpha m} r^{(j)}_{mm'}(\beta) e^{-\mathrm{i}\,\gamma m'}$$

となる (ウィグナーの公式).

和 $\sum_\ell$ は階乗の引数が正または 0 (但し $0! = 1$ とする) になるようなすべての ℓ について和をとる.

具体的な表現行列

$(2j+1)$-次元ユニタリ行列:

$$R^{(j)}(\alpha,\beta,\gamma) = \left(R^{(j)}_{mm'}(\alpha,\beta,\gamma)\right) = \left(e^{-\mathrm{i}\,\alpha m} r^{(j)}_{mm'}(\beta)\,e^{-\mathrm{i}\,\gamma m'}\right)$$

<u>$j = {}^1\!/_2$ の場合</u> $\quad r^{(1/2)}(\beta) = \left(r^{(1/2)}_{mm'}(\beta)\right) = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$

<u>$j = 1$ の場合</u> $\quad r^{(1)}(\beta) = \left(r^{(1)}_{mm'}(\beta)\right) = \begin{pmatrix} c^2 & -\sqrt{2}cs & s^2 \\ \sqrt{2}cs & 2c^2-1 & -\sqrt{2}cs \\ s^2 & \sqrt{2}cs & c^2 \end{pmatrix}$

<u>$j = {}^3\!/_2$ の場合</u> $\quad r^{(3/2)}(\beta) = \left(r^{(3/2)}_{mm'}(\beta)\right) = \begin{pmatrix} c^3 & -\sqrt{3}c^2s & \sqrt{3}cs^2 & -s^3 \\ \sqrt{3}c^2s & -2cs^2+c^3 & s^3-2c^2s & \sqrt{3}cs^2 \\ \sqrt{3}cs^2 & -s^3+2c^2s & -2cs^2+c^3 & -\sqrt{3}c^2s \\ s^3 & \sqrt{3}cs^2 & \sqrt{3}c^2s & c^3 \end{pmatrix}$

$$c = \cos(\beta/2)\ ,\ s = \sin(\beta/2)$$

行列やベクトルの成分を m, m' を指標として書くとき
添え字の m と m' は, j から $-j$ まで大きい値から小さい値に 1 つずつ減る順番に並べて記すことにする.

多成分量子ウォーク模型の定義

時刻 t で $x \in \mathrm{Z}$ にいるウォーカーの量子状態

$$\begin{pmatrix} \Psi_{1/2}(x,t) \\ \Psi_{-1/2}(x,t) \end{pmatrix} = \int_{-\pi}^{\pi}\frac{dk}{2\pi}\left[\begin{pmatrix} e^{\mathrm{i}\,k} & 0 \\ 0 & e^{-\mathrm{i}\,k} \end{pmatrix}\begin{pmatrix} c\,e^{-\mathrm{i}(\alpha+\gamma)/2} & -s\,e^{-\mathrm{i}(\alpha-\gamma)/2} \\ s\,e^{\mathrm{i}(\alpha-\gamma)/2} & c\,e^{\mathrm{i}(\alpha+\gamma)/2} \end{pmatrix}\right]^t \begin{pmatrix} q_{1/2} \\ q_{-1/2} \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} \Psi^{(j)}_j(x,t) \\ \Psi^{(j)}_{j-1}(x,t) \\ \vdots \\ \Psi^{(j)}_{-j}(x,t) \end{pmatrix} = \int_{-\pi}^{\pi}\frac{dk}{2\pi}\left[\begin{pmatrix} e^{2\,\mathrm{i}\,jk} & & & \\ & e^{2\,\mathrm{i}(j-1)k} & & \\ & & \ddots & \\ & & & e^{-2\,\mathrm{i}\,jk} \end{pmatrix} R^{(j)}(\alpha,\beta,\gamma)\right]^t \begin{pmatrix} q_j \\ q_{j-1} \\ \vdots \\ q_{-j} \end{pmatrix}$$

2 成分量子ビット $\quad\Rightarrow\quad$ $(2j+1)$-成分量子ビット

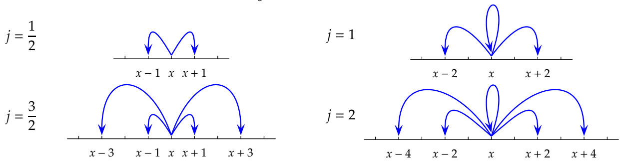
$$\mathrm{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$$

実空間での 1 ステップ

$$\Psi^{(j)}_m(x,t+1) = \sum_{m'=-j}^{j} R^{(j)}_{mm'}\Psi^{(j)}_{m'}(x+2m,t)$$

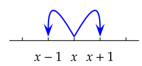
$$\Psi^{(j)}(x,0) = \delta_{x0}\phi^{(j)}_0$$

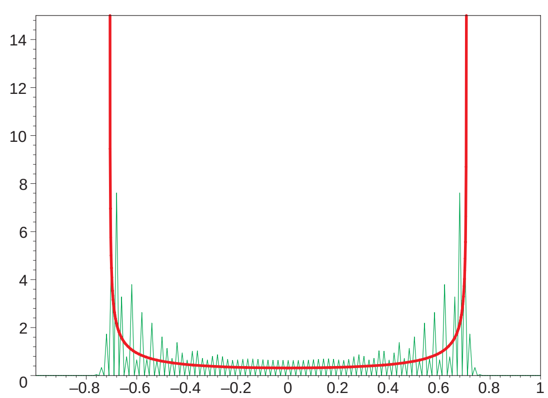
$$\phi^{(j)}_0 = \begin{pmatrix} q_j \\ q_{j-1} \\ \vdots \\ q_{-j} \end{pmatrix} \quad \text{但し} \sum_{m=-j}^{j}|q_m|^2 = 1$$



§4 シミュレーションの結果

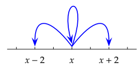
2 状態 $\quad \phi_0 = \frac{1}{2}\begin{pmatrix} 1+\mathrm{i} \\ 1-\mathrm{i} \end{pmatrix}, \quad R^{(1/2)} = \frac{\mathrm{i}}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

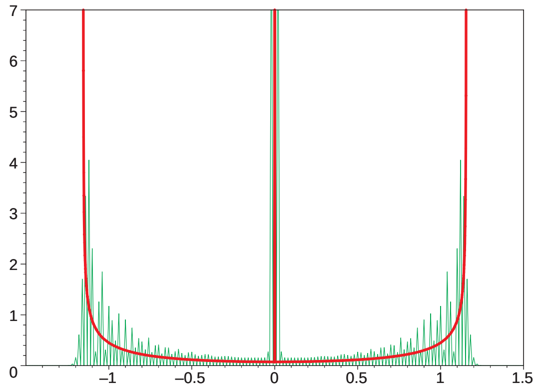




3 状態

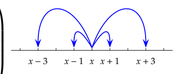
$$\phi_0 = \frac{1}{\sqrt{6}}\begin{pmatrix}1-\mathrm{i}\\1+\mathrm{i}\\1-\mathrm{i}\end{pmatrix}, \quad R^{(1)} = \frac{1}{3}\begin{pmatrix}-1 & -2 & -2\\-2 & -1 & 2\\-2 & 2 & -1\end{pmatrix}$$

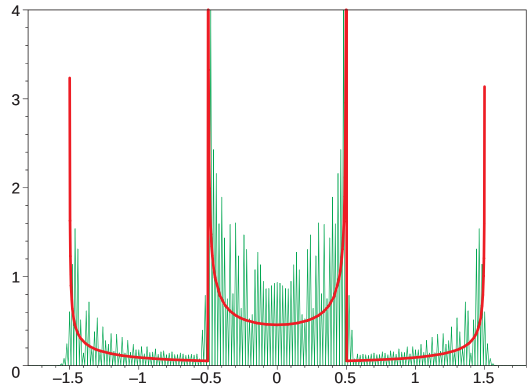




4 状態

$$\phi_0 = \frac{1}{2\sqrt{5}}\begin{pmatrix}1+3\,\mathrm{i}\\0\\0\\-3+\mathrm{i}\end{pmatrix}, \quad R^{(3/2)} = \frac{\mathrm{i}}{8}\begin{pmatrix}1 & 3 & 3\sqrt{3} & 3\sqrt{3}\\3 & 5 & \sqrt{3} & -3\sqrt{3}\\3\sqrt{3} & \sqrt{3} & -5 & 3\\3\sqrt{3} & -3\sqrt{3} & 3 & -1\end{pmatrix}$$





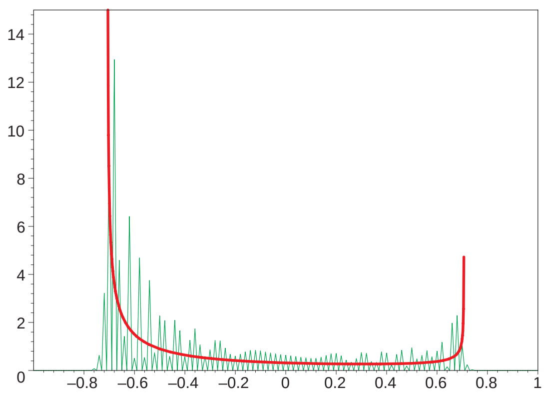
2 状態

symmetric

$$\phi_0 = \frac{1}{2}\begin{pmatrix}1+\mathrm{i}\\1-\mathrm{i}\end{pmatrix}$$

asymmetric

$$\phi_0 = \frac{1}{2}\begin{pmatrix}1+\mathrm{i}\\1+\mathrm{i}\end{pmatrix}$$



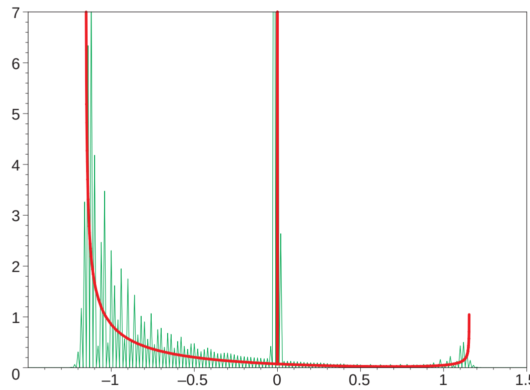
3 状態

symmetric

$$\phi_0 = \frac{1}{\sqrt{6}}\begin{pmatrix}1-\mathrm{i}\\1+\mathrm{i}\\1-\mathrm{i}\end{pmatrix}$$

asymmetric

$$\phi_0 = \frac{1}{\sqrt{6}}\begin{pmatrix}1-\mathrm{i}\\1-\mathrm{i}\\1-\mathrm{i}\end{pmatrix}$$



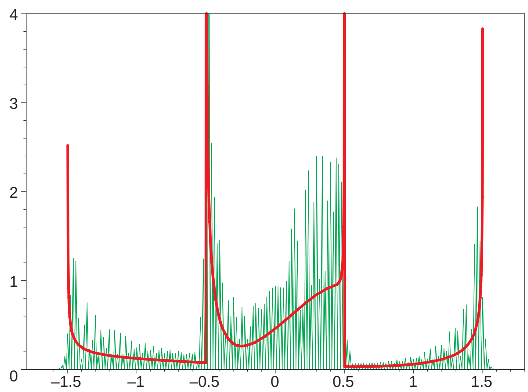
4 状態

symmetric

$$\phi_0 = \frac{1}{2\sqrt{5}}\begin{pmatrix}1+3\,\mathrm{i}\\0\\0\\-3+\mathrm{i}\end{pmatrix}$$

asymmetric

$$\phi_0 = \frac{1}{2\sqrt{5}}\begin{pmatrix}1+3\,\mathrm{i}\\0\\0\\-3-\mathrm{i}\end{pmatrix}$$



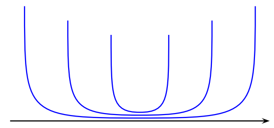
§5 一般化された極限分布

1) $(2j+1)$-成分量子ウォークの擬速度の分布は長時間極限でモーメント収束する.

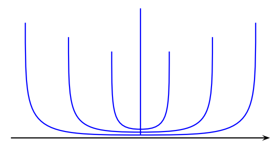
$$\lim_{t\to\infty}\left\langle\left(\frac{X_t^{(j)}}{t}\right)^r\right\rangle = \int_{-\infty}^{\infty} dy\, y^r \nu^{(j)}(y), \quad r = 0, 1, 2, \cdots$$

2) 極限分布は

$$\nu^{(j)}(y) = \sum_{m:0<m\leq j}\frac{1}{2m}\mu\left(\frac{y}{2m};\cos\frac{\beta}{2}\right)\mathcal{M}^{(j,m)}\left(\frac{y}{2m}\right) + \mathbf{1}_{\{(2j+1)\,\text{is odd}\}}\Delta^{(j)}\delta(y)$$



qubit の成分数 $(2j+1)$ = 偶数
今野分布の重ね合せ



qubit の成分数 $(2j+1)$ = 奇数
今野分布の重ね合せ
+ デルタ関数 (局在成分)

今野分布の重ね合わせの重み

$$\lim_{t\to\infty}\left\langle\left(\frac{X_t^{(j)}}{t}\right)^r\right\rangle = \int_{-\infty}^{\infty} dy\, y^r \nu^{(j)}(y)$$

$$\nu^{(j)}(y) = \sum_{m:0<m\le j}\frac{1}{2m}\mu\left(\frac{y}{2m};\cos\frac{\beta}{2}\right)\mathcal{M}^{(j,m)}\left(\frac{y}{2m}\right) + \mathbf{1}_{\{(2j+1)\text{ is odd}\}}\Delta^{(j)}\delta(y)$$

- $\left.\begin{array}{l}\alpha,\beta,\gamma \text{ パラメータ}\\ \text{初期 qubit}\end{array}\right\}$ を情報として持っている
- $\mathcal{M}^{(j,m)}(x)$ は x の $2j$-次多項式である

$j = 1/2$ の場合 (2 成分模型)

$$\mathcal{M}^{(1/2,1/2)}(x) = 1 + \mathcal{M}_1^{(1/2,1/2)}x$$

$$\mathcal{M}_1^{(1/2,1/2)} = -\{|q_{1/2}|^2 - |q_{-1/2}|^2\} + 2\tau\,\mathrm{Re}\Big\{q_{1/2}\overline{q}_{-1/2}e^{-\mathrm{i}\gamma}\Big\}$$

$j = 1$ の場合 (3 成分模型)

$$\mathcal{M}^{(1,1)}(x) = \mathcal{M}_0^{(1,1)} + \mathcal{M}_1^{(1,1)}x + \mathcal{M}_2^{(1,1)}x^2$$

$$\mathcal{M}_0^{(1,1)} = \frac{1}{2}\{|q_1|^2 + 2|q_0|^2 + |q_{-1}|^2\} - \mathrm{Re}\Big\{q_1\overline{q}_{-1}e^{-2\mathrm{i}\gamma}\Big\}$$

$$\mathcal{M}_1^{(1,1)} = -\{|q_1|^2 - |q_{-1}|^2\} + \sqrt{2}\tau\,\mathrm{Re}\Big\{(q_1\overline{q}_0 + q_0\overline{q}_{-1})e^{-\mathrm{i}\gamma}\Big\}$$

$$\mathcal{M}_2^{(1,1)} = \frac{1}{2}\{|q_1|^2 - 2|q_0|^2 + |q_{-1}|^2\} - \sqrt{2}\tau\,\mathrm{Re}\Big\{(q_1\overline{q}_0 - q_0\overline{q}_{-1})e^{-\mathrm{i}\gamma}\Big\} + (1+2\tau^2)\,\mathrm{Re}\Big\{q_1\overline{q}_{-1}e^{-2\mathrm{i}\gamma}\Big\}$$

$$\Delta^{(1)} = 1 - \left\{\mathcal{M}_0^{(1,1)} + \left(1 - \sin\frac{\beta}{2}\right)\mathcal{M}_2^{(1,1)}\right\}$$

$j = 3/2, m = 3/2$ の場合 (4 成分模型)

$$\mathcal{M}^{(3/2,3/2)}(x) = \mathcal{M}_0^{(3/2,3/2)} + \mathcal{M}_1^{(3/2,3/2)}x + \mathcal{M}_2^{(3/2,3/2)}x^2 + \mathcal{M}_3^{(3/2,3/2)}x^3$$

$$\mathcal{M}_0^{(3/2,3/2)} = \frac{1}{4}\Big\{|q_{3/2}|^2 + 3|q_{1/2}|^2 + 3|q_{-1/2}|^2 + |q_{-3/2}|^2\Big\} - \frac{\sqrt{3}}{2}\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{-1/2} + q_{1/2}\overline{q}_{-3/2})e^{-2\mathrm{i}\gamma}\Big\}$$

$$\begin{aligned}\mathcal{M}_1^{(3/2,3/2)} = &-\frac{3}{4}\Big\{|q_{3/2}|^2 + |q_{1/2}|^2 - |q_{-1/2}|^2 - |q_{-3/2}|^2\Big\} - \frac{3}{2}\tau\,\mathrm{Re}\Big\{q_{3/2}\overline{q}_{-3/2}e^{-3\mathrm{i}\gamma} - q_{1/2}\overline{q}_{-1/2}e^{-\mathrm{i}\gamma}\Big\}\\ &+ \frac{\sqrt{3}}{2}\tau\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{1/2} + q_{-1/2}\overline{q}_{-3/2})e^{-\mathrm{i}\gamma}\Big\} + \frac{\sqrt{3}}{2}\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{-1/2} - q_{1/2}\overline{q}_{-3/2})e^{-2\mathrm{i}\gamma}\Big\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_2^{(3/2,3/2)} = &\frac{3}{4}\Big\{|q_{3/2}|^2 - |q_{1/2}|^2 - |q_{-1/2}|^2 + |q_{-3/2}|^2\Big\} - \sqrt{3}\tau\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{1/2} - q_{-1/2}\overline{q}_{-3/2})e^{-\mathrm{i}\gamma}\Big\}\\ &+ \frac{\sqrt{3}}{2}(1+2\tau^2)\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{-1/2} + q_{1/2}\overline{q}_{-3/2})e^{-2\mathrm{i}\gamma}\Big\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_3^{(3/2,3/2)} = &-\frac{1}{4}\Big\{|q_{3/2}|^2 - 3|q_{1/2}|^2 + 3|q_{-1/2}|^2 - |q_{-3/2}|^2\Big\} + \frac{1}{2}\tau(3+4\tau^2)\,\mathrm{Re}\Big\{q_{3/2}\overline{q}_{-3/2}e^{-3\mathrm{i}\gamma}\Big\}\\ &- \frac{3}{2}\tau\,\mathrm{Re}\Big\{q_{1/2}\overline{q}_{-1/2}e^{-\mathrm{i}\gamma}\Big\} + \frac{\sqrt{3}}{2}\tau\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{1/2} + q_{-1/2}\overline{q}_{-3/2})e^{-\mathrm{i}\gamma}\Big\}\\ &- \frac{\sqrt{3}}{2}(1+2\tau^2)\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{-1/2} - q_{1/2}\overline{q}_{-3/2})e^{-2\mathrm{i}\gamma}\Big\}\end{aligned}$$

$j = 3/2, m = 1/2$ の場合 (4 成分模型)

$$\mathcal{M}^{(3/2,1/2)}(x) = \mathcal{M}_0^{(3/2,1/2)} + \mathcal{M}_1^{(3/2,1/2)}x + \mathcal{M}_2^{(3/2,1/2)}x^2 + \mathcal{M}_3^{(3/2,1/2)}x^3$$

$$\mathcal{M}_0^{(3/2,1/2)} = \frac{1}{4}\Big\{3|q_{3/2}|^2 + |q_{1/2}|^2 + |q_{-1/2}|^2 + 3|q_{-3/2}|^2\Big\} + \frac{\sqrt{3}}{2}\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{-1/2} + q_{1/2}\overline{q}_{-3/2})e^{-2\mathrm{i}\gamma}\Big\}$$

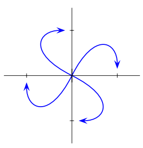
$$\begin{aligned}\mathcal{M}_1^{(3/2,1/2)} = &-\frac{1}{4}\Big\{3|q_{3/2}|^2 - 5|q_{1/2}|^2 + 5|q_{-1/2}|^2 - 3|q_{-3/2}|^2\Big\} + \frac{9}{2}\tau\,\mathrm{Re}\Big\{q_{3/2}\overline{q}_{-3/2}e^{-3\mathrm{i}\gamma}\Big\}\\ &- \frac{1}{2}\tau\,\mathrm{Re}\Big\{q_{1/2}\overline{q}_{-1/2}e^{-\mathrm{i}\gamma}\Big\} + \frac{\sqrt{3}}{2}\tau\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{1/2} + q_{-1/2}\overline{q}_{-3/2})e^{-\mathrm{i}\gamma}\Big\}\\ &- \frac{3\sqrt{3}}{2}\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{-1/2} - q_{1/2}\overline{q}_{-3/2})e^{-2\mathrm{i}\gamma}\Big\}\end{aligned}$$

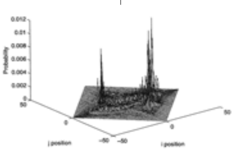
$$\begin{aligned}\mathcal{M}_2^{(3/2,1/2)} = &-\frac{3}{4}\Big\{|q_{3/2}|^2 - |q_{1/2}|^2 - |q_{-1/2}|^2 + |q_{-3/2}|^2\Big\} + \sqrt{3}\tau\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{1/2} - q_{-1/2}\overline{q}_{-3/2})e^{-\mathrm{i}\gamma}\Big\}\\ &- \frac{\sqrt{3}}{2}(1+2\tau^2)\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{-1/2} + q_{1/2}\overline{q}_{-3/2})e^{-2\mathrm{i}\gamma}\Big\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_3^{(3/2,1/2)} = &\frac{3}{4}\Big\{|q_{3/2}|^2 - 3|q_{1/2}|^2 + 3|q_{-1/2}|^2 - |q_{-3/2}|^2\Big\} - \frac{3}{2}\tau(3+4\tau^2)\,\mathrm{Re}\Big\{q_{3/2}\overline{q}_{-3/2}e^{-3\mathrm{i}\gamma}\Big\}\\ &+ \frac{9}{2}\tau\,\mathrm{Re}\Big\{q_{1/2}\overline{q}_{-1/2}e^{-\mathrm{i}\gamma}\Big\} - \frac{3\sqrt{3}}{2}\tau\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{1/2} + q_{-1/2}\overline{q}_{-3/2})e^{-\mathrm{i}\gamma}\Big\}\\ &+ \frac{3\sqrt{3}}{2}(1+2\tau^2)\,\mathrm{Re}\Big\{(q_{3/2}\overline{q}_{-1/2} - q_{1/2}\overline{q}_{-3/2})e^{-2\mathrm{i}\gamma}\Big\}\end{aligned}$$

§6 今後の課題

- $\mathcal{M}^{(j,m)}$ の一般的な表式
  $j = 1/2, 1, 3/2$ は完全に求まった
  ↓
  一般形は?
- 1 次元 → 2 成分, 3 成分, 4 成分,$\cdots$
  ↓
  2 次元 (正方格子) → 4 成分,$\cdots$
  ↓
  3 次元 (立方格子) → 6 成分,$\cdots$





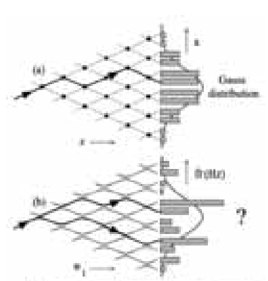
S. E. Venagas-Andraca, *et al.*
New J. Phys. **7**, 221 (2005)

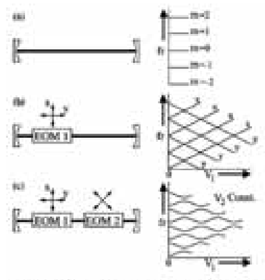
- N. Inui, Y. Konishi, and N. Konno, *PRA* **69**, 052323 (2004)
- G. Grimmett, S. Janson, and P. F. Scudo, *PRE* **69**, 026119 (2004)

Wigner formula of rotation matrices and quantum walks
T. Miyazaki, M. Katori, and N. Konno (quant-ph/0611022)

APPENDIX A: 実験について

Bouwmeester *et al.* : Optical Galton board, PRA **61**, 013410 (1999)





Oka, Konno, Arita and Aoki : Breakdown of an Electric-Field Driven System: A Mapping to a Quantum Walk, PRL **94**, 100602 (2005).

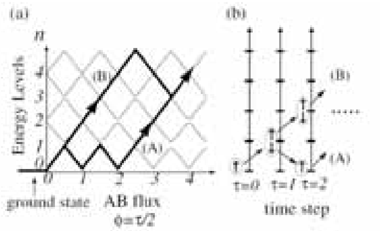


FIG. 1. (a) Idealized energy levels of an electron system on a ring plotted against the AB flux $\phi = FLt/h$, which increases linearly with time after the electric field F is turned on at $t = 0$. Two paths (A and B) for the Landau-Zener transition among neighboring levels are shown. (b) A mapping to a quantum walk, where the energy levels are mapped to sites of the qubits and the ground state to a reflecting boundary.

APPENDIX B: 計算の詳細

$\hat{\Psi}^{(j)}(k,t)$ の表式

$$V^{(j)}(k) = R^{(j)}(\phi(k),\theta(k),0)R^{(j)}(-p(k),0,0)[R^{(j)}(\phi(k),\theta(k),0)]^\dagger$$

- $\mathbf{v}_m^{(j)}(k) : R^{(j)}(\phi(k),\theta(k),0)$ の m 番目の列ベクトル
- $C_m^{(j)}(k) \equiv [\mathbf{v}_m^{(j)}(k)]^\dagger \phi_0^{(j)}$

$$\hat{\Psi}^{(j)}(k,t) = \left(V^{(j)}(k)\right)^t \phi_0^{(j)} = \sum_{m=-j}^{j} e^{\mathrm{i}\,tmp(k)} \mathbf{v}_m^{(j)}(k) C_m^{(j)}(k)$$

$$[R^{(j)}]^\dagger = [R^{(j)}]^{-1} \quad \Leftrightarrow \quad [\mathbf{v}_m^{(j)}(k)]^\dagger \mathbf{v}_{m'}^{(j)}(k) = \delta_{mm'}$$

$$[\hat{\Psi}^{(j)}(k,t)]^\dagger \left(\mathrm{i}\frac{d}{dk}\right)^r \hat{\Psi}^{(j)}(k,t) = \sum_{m=-j}^{j} \left(-m\frac{dp(k)}{dk}\right)^r |C_m^{(j)}(k)|^2 t^r + \mathcal{O}(t^{r-1})$$

擬速度 のモーメント

$$\left\langle (X_t^{(j)})^r \right\rangle = \int_{-\infty}^{\infty} \frac{dk}{2\pi} [\hat{\Psi}^{(j)}(k,t)]^\dagger \left(\mathrm{i}\frac{d}{dk}\right)^r \hat{\Psi}^{(j)}(k,t)$$

$r = 1, 2, 3, \cdots$ としたときの r-次モーメントは

$$\lim_{t\to\infty} \left\langle \left(\frac{X_t^{(j)}}{t}\right)^r \right\rangle = \sum_{m:0<m\le j} I_m^{(j)}(r)$$

$$I_m^{(j)}(r) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left\{(-1)^r |C_m^{(j)}(k)|^2 + |C_{-m}^{(j)}(k)|^2\right\} \left(m\frac{dp(k)}{dk}\right)^r$$

j が半奇数なら $m = 1/2, 3/2, \cdots, j$
j が正の整数なら $m = 1, 2, \cdots, j$

軌道平面

$$\mathbf{p}(k) = (p_1(k), p_2(k), p_3(k))$$

$$p_1(k) = p(k)\sin\theta(k)\cos\phi(k)$$
$$p_2(k) = p(k)\sin\theta(k)\sin\phi(k)$$
$$p_3(k) = p(k)\cos\theta(k)$$

$$\hat{\mathbf{e}}_1 = (-\sin\gamma, -\cos\gamma, 0)$$
$$\hat{\mathbf{e}}_2 = (\sin\frac{\beta}{2}\cos\gamma, -\sin\frac{\beta}{2}\sin\gamma, -\cos\frac{\beta}{2})$$
$$\hat{\mathbf{e}}_3 = (\cos\frac{\beta}{2}\cos\gamma, -\cos\frac{\beta}{2}\sin\gamma, \sin\frac{\beta}{2})$$

$$\mathbf{p}(k) \perp \hat{\mathbf{e}}_3 \quad \text{for all} \quad k \in [-\pi, \pi)$$

軌道平面上 の極座標 $(p(k), \chi)$:

$$\cos\chi \equiv \hat{\mathbf{p}}(k)\cdot\hat{\mathbf{e}}_1,\ \hat{\mathbf{p}}(k) = \mathbf{p}(k)/p(k) \quad \Rightarrow \quad \underline{\tan\frac{p(k)}{2} = \tan\frac{\beta}{2}\frac{1}{\cos\chi}}$$

$\chi \to y$ と変数変換

$$\lim_{t\to\infty} \left\langle \left(\frac{X_t^{(j)}}{t}\right)^r \right\rangle = \int_{-\infty}^{\infty} dy\, y^r \nu^{(j)}(y), \quad r = 0, 1, 2, \cdots$$

$$\nu^{(j)}(y) = \sum_{m:0<m\le j} \frac{1}{2m}\mu\left(\frac{y}{2m}; \cos\frac{\beta}{2}\right) \mathcal{M}^{(j,m)}\left(\frac{y}{2m}\right) + \mathbf{1}_{\{(2j+1)\ \text{is odd}\}} \Delta^{(j)}\delta(y)$$