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Analysis of canopy-gap structures of forests by thermal equilibrium correlation equalities

Toshiro Takamatsu and Makoto Katori
Department of Physics, Faculty of Science and Engineering, Chuo University, Kasuga, Bunkyo-ku, Tokyo 112-8551
Fax: 81-3-3817-1792, e-mail: takamatu@phys.chuo-u.ac.jp, katori@phys.chuo-u.ac.jp

The uppermost branchy layer of trees is called canopy. Analysis of canopy-gap sizes is important to characterize the forest dynamics. Kubo, Iwasa and Furumoto (KIF) introduced a continuous-time stochastic model on a square lattice for canopy-gap dynamics and Katori *et al.* showed the condition that the stationary state of the KIF model is given by the Ising-Gibbs state. In the present paper we propose a new method to estimate the values of parameters for the Ising-Gibbs state so that the real forest data are well approximated by it. In this new method we use the thermal equilibrium correlation equalities and it is not necessary to perform Monte Carlo simulation in order to adjust the parameters. Applications to the real forest data on Barro Colorado Island, Panama, and in Ogawa Forest Reserve, Japan, show the validity of the present method. All results support that the real forests are not exactly critical but nearly critical in the canopy-gap distributions.
Key words: canopy gaps, forest dynamics, Ising-Gibbs measure, thermal equilibrium correlation equalities, criticality

1. INTRODUCTION

Canopy is the uppermost spreading branchy layer of trees. In data of a census of a forest plot [1, 2, 3], it is divided into subplots, each area of which is usually 5×5 m. If height of trees in a subplot is higher than a certain threshold (*e.g.* 20 m), the subplot is called a canopy site. If not, it is called a canopy-gap site. Figure 1 shows the configuration of gap sites in a neotropical forest on Barro Colorado Island (BCI), Panama, in 1983 [1, 7], where a black dot denotes a canopy-gap site. We can define canopy-gap cluster in such a digitized map and measure the distribution of cluster sizes in real forest data. As shown in Fig.2, the distribution seems to obey a power-law. Such power-law distributions of canopy-gap sizes have been reported by many ecologists [1, 2, 3] and have attracted much attention of statistical physicists and theoretical biologists [4, 5, 6, 7, 8, 9], since they imply that the self-organized criticality[10, 11] is realized in real forests.

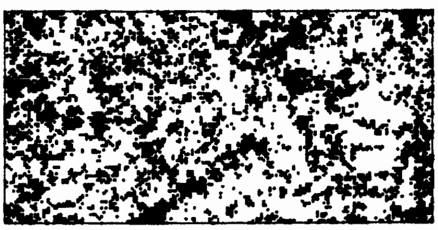

Fig. 1: 1000 × 500 m digitized map of the neotoropical forest on Baro Colorado Island, Panama, in 1983. Gap sites are plotted by black dots.

Kubo, Iwasa and Furumoto (KIF) introduced a continuous-time stochastic model on a square lattice for canopy-gap dynamics [7]. Then Katori *et al.* showed the condition that the stationary state of the KIF model is given by the Ising-Gibbs state [8]. The canopy-gap dynamics has been studied by using an Ising model which is the simplest model for a magnet. Fitting of the Ising-Gibbs state to the real data of a neotropical forest on BCI, Panama, and of a deciduous forest in Ogawa Forest Reserve (OFR), Japan, show that canopy-gap structures of the forests are not exactly critical but nearly critical [8, 9].

2. A METHOD TO ANALYZE FOREST DATA

2.1 Ising Model

We consider an Ising model on a finite square lattice. A spin on each site i is expressed by σ_i and $\sigma_i = -1$ or $+1$. Here we consider the following probability distribution with two parameters K and

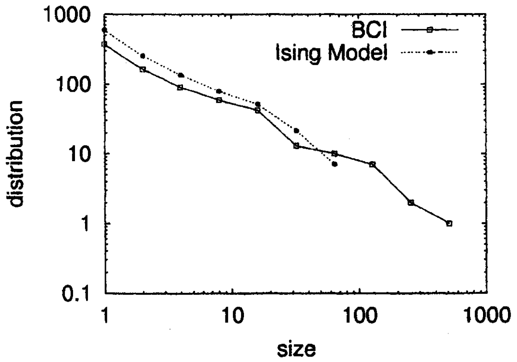

Fig. 2: Canopy-gap size distribution for BCI and the cluster-size distribution of down spins in the Ising-Gibbs state with $K = 0.350$ and $h = 0.043$.

h,

$$\nu\{\sigma_i\} = \frac{1}{Z}\exp\left[K\sum_{i,j:|i-j|=1}\sigma_i\sigma_j + h\sum_i \sigma_i\right], \quad (1)$$

where Z is the partition function. We will simply refer to (1) as the Ising-Gibbs state in this paper. We will write the expectation of a physical quantity $Q\{\sigma_i\}$ in the Ising-Gibbs state as $\langle Q\{\sigma_i\}\rangle$. The critical point in parameter space (K,h) is given as (K_c,0) with $K_c = \log(1+\sqrt{2})/2 = 0.4406...$ and there occurs a continuous phase transition with critical phenomena at this critical point.

2.2 Thermal Equilibrium Correlation Equalities

In the Ising-Gibbs state, correlation equalities

$$\langle\sum_{i=1}^{n}\sigma_i\rangle = \frac{1}{n}\sum_{j=1}^{n}\langle\Big(\sum_{i:i\neq j}\sigma_i\Big)\tanh E_j\rangle, \quad (2)$$

should be satisfied for an arbitrary integer n [12], where

$$E_j = h + K\sum_{k:|k-j|=1}\sigma_k. \quad (3)$$

In the present paper, we call such equalities *the thermal equilibrium correlation equalities*. In particular we consider the n=1, 2 cases and have

$$\langle\sigma_i\rangle = \langle\tanh E_i\rangle, \quad (4)$$

$$\langle\sigma_i\sigma_{i+1}\rangle = \frac{1}{2}\langle\sigma_{i+1}\tanh E_i\rangle + \frac{1}{2}\langle\sigma_i\tanh E_{i+1}\rangle, \quad (5)$$

where i and $i+1$ are the nearest-neighbor pair of sites.

Now we propose a new method to estimate the values of K and h of the Ising-Gibbs state so that it can approximate the real forest data. It should be noted that Monte Carlo simulations should be performed to search the suitable values of parameters K and h in our previous method [8, 9]. Here we show that, if we use the thermal equilibrium correlation equalities, we can estimate the values of K and h directly form the real data of canopy-gap configurations without Monte Carlo simulations.

We consider two correlation equalities (4) and (5) as simultaneous equations. We numerically solve these equalities as follows.

Assume that a digitized data of forest such as Fig.1 is given. We regard a canopy site as an up spin $\sigma_i = 1$ and a canopy-gap site as a down spin $\sigma_i = -1$. Let N be the total number of sites and N_+ (resp. N_-) be the total number of *up* (resp. *down*) *spins* in the forest data. Then the expectation $\langle\sigma_i\rangle$ is calculated as

$$\langle\sigma_i\rangle = \frac{N_+ - N_-}{N}. \quad (6)$$

By the similar procedure, the expectation of nearest-neighbor spin-pair correlation $\langle\sigma_i\sigma_i+1\rangle$ can be calculated for the forest data. (i) Assume that certain values of K and h are given. (ii) The averaged values $\langle\tanh E_i\rangle$ and $\langle\sigma_i\tanh E_{i+1}\rangle$ are calculated for the forest data, where E_j is given by (3). (iii) If these quantities satisfy the correlation (4), we plot a dot at (K,h) on the parameter plane. (iv) The some procedure is done for the correlation equality (5). (v) Change the values of K and h and repeat these procedures from (ii) to (iv). Thus two lines, which express the set of parameters satisfying (4) and (5), respectively, are drawn on the K-h plane as shown in Fig.3. An intersection of these two lines gives the values of K and h for the given forest data.

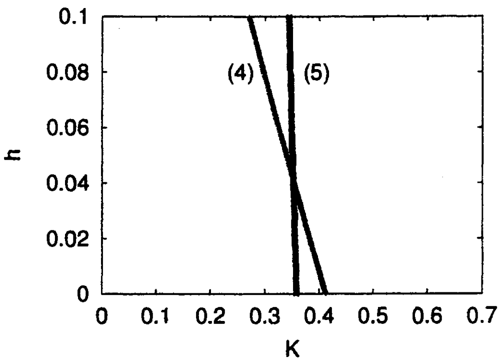

Fig. 3: Two lines satisfying the correlation equalities(4) and (5), respectively, for BCI in 1983. An intersection of these two lines gives the estimated values $K = 0.350$ $h = 0.043$ for the given forest data.

3. RESULTS OF ANALYSIS

3.1 Analysis of Barro Colorado Island

Hubbel and Foster studied a neotropical forest on Barro Colorado Island (BCI), Panama, [1, 13]. By the present new method using the correlation equalities, Fig.3 is drawn. From an intersection of two lines there, we have an Ising-Gibbs state (1) with $K = 0.350$ and $h = 0.043$.

We define $f_f(s)$ as a number of gap clusters whose size is s in the forest and $f_{IG}(s)$ as that of down-spin clusters in the Ising-Gibbs state. Figure 2 shows the log-log plot of $f_f(s)$ for BCI and $f_{IG}(s)$ with $K = 0.350$ and $h = 0.043$ evaluated by the Monte Carlo simulations. From Fig.2, we conclude that $f_f(s)$ is approximated well by $f_{IG}(s)$ over wide range of scales.

3.2 Analysis of Ogawa Forest Reserve

Tanaka and Nakashizuka and their coworkers studied a deciduous forest in Ogawa Forest Reserve (OFR), Japan, at five-year intervals from 1976 to 1991 by aerial photographs [3, 14, 15, 16]. A total area of the forest plot is 555×455 m and each area of subplot is 5×5 m. Then a lattice size for OFR is 111×91. Tanaka and Nakashizuka have allowed us to use their original data of height distributions of canopy. First we define gap sites by the subplots where there is no canopy taller than 15 m according to ref.1.

The values of K and h in OFR for 1976, 1981, 1986 and 1991 are evaluated by the present method using thermal equilibrium correlation equalities and results are shown in Table 1 and Fig.4. It is interesting that evaluated (K,h)'s are always located near the critical point $(K_c,0)$.

Table 1: List of the parameters K and h which are chosen to satisfy the correlation equalities for BCI in 1983 and for OFR in 1976, 1981, 1986 and 1991.

Data	K	h
BCI(1983)	0.350	0.043
OFR(1976)	0.414	0.052
OFR(1981)	0.428	0.039
OFR(1986)	0.410	0.058
OFR(1991)	0.559	-0.023

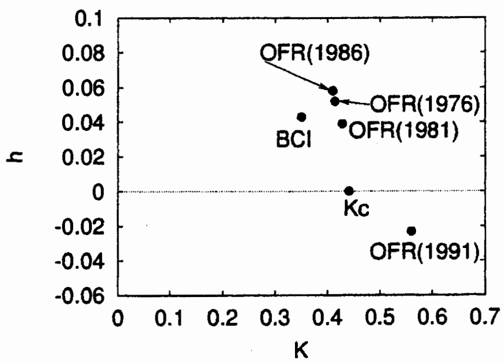

Fig. 4: Plots of the results in Table 1 on a parameter plane (K,h). All plots are located near the critical point $(K_c,0)$.

3.3 Analysis of OFR for Various Thresholds

A datum on each site is given by a canopy height itself for OFR in 1976, 1981, 1986 and 1991. Now we redefine canopy-gap sites by changing the threshold height from the value 15 m. For example, if we increase the value of threshold height, the number of canopy-gap sites increases.

The results are shown in Table 2 and Fig.5. It should be noted that in Fig.5 the values of K and h surround a critical point $(K_c,0)$.

Table 2: List of the parameters K and h for OFR in 1986 ,which are chosen to satisfy the correlation equalities on the configuration of canopy-gap sites for each value of threshold.

Threshold [m]	K	h
12	0.521	-0.010
13	0.474	0.030
14	0.411	0.104
15	0.410	0.058
16	0.382	0.064
17	0.367	0.051
18	0.365	0.024
19	0.371	-0.005
20	0.397	-0.037
21	0.421	-0.044

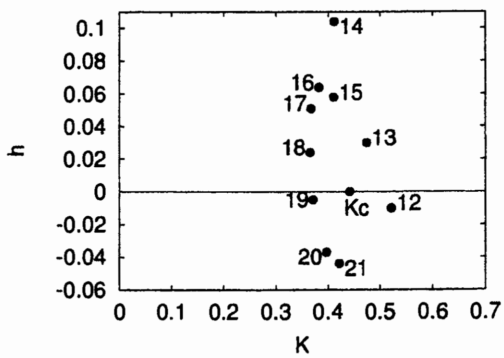

Fig. 5: Plots of the results in Table 2 on a parameter plane (K,h). The numbers associated with dots indicate the values of threshold height [m]. All plots surround a critical point $(K_c,0)$.

4. CONCLUSION

In the present paper, we proposed a new method to analyze real data of forests by the Ising-Gibbs states. In this method, the parameters K and h are evaluated by using the thermal equilibrium correlation equalities. We analyze two forests on Barro Colorado Island, Panama, and in Ogawa Forest Reserve, Japan. All results support the statement that the forest data are well described by the Ising-Gibbs states and the states are not exactly critical but nearly critical. Ecological meaning of (near) criticality and physical mechanism to maintain the forests in (nearly) critical states are not yet clarified.

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