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On the Drossel-Schwabl limit in a forest fire model

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In the forest fire model of Drossel and Schwabl, lightning with probability f, deterministic fire spreading, and growth of tree with probability q are proceeding in the same time scale. In such a parallel updated algorithm, system shows criticality only in the double limit $q \to 0$ and $f/q \to 0$, which we call the Drossel-Schwabl limit (DS limit). A modified model of forest fire is proposed, in which the lightning and growing are prohibited during fire spreading and they can proceed only after any fire has ceased: time scales are separated on algorithm. We define the cluster of green-tree sites and study the percolation transitions. For small values of f and f/q, oscillation phases with period n are observed at least $n = 2$ and 3, in which the percolation probability of green-tree sites has positive value only at every n-th step. An envelope of a series of phase boundaries of oscillation phases with period $n = 2, 3, 4, \cdots$ in the (f,q)-phase diagram will give a physical realization of DS limit.

Key words: forest fire models, forest dynamics, self-organized criticality, percolation, oscillation phases, Drossel-Schwabl limit

1. INTRODUCTION

Large scale disturbances, such as, fires, typhoons, avalanches and so on, play important roles in the formation of spatial structures of forests and have been studied by many ecologists [1, 2]. In particular forest fires have been studied for a long time. For example, extensive set of time series data are reported on forest fires in the boreal forest regions of Europe and North America, and simulation models are proposed [3]. On the other hand, from the view point of statistical physics, criticality observed in the real forest data is very interesting [4, 5, 6, 7, 8]. For example, the cluster-size distribution of canopy-gap is well described by power-law distribution in the forest on Barro Colorado Island, Panama [9, 10], and in Ogawa forest reserve, Japan [11]. Forest dynamics will provide interesting examples of self-organized criticality [12, 13].

2. THE DROSSEL-SCHWABL LIMIT

In 1990 Bak, Chen and Tang introduced a simple forest fire model [14]. In 1992 this model was modified by Drossel and Schwabl and a stochastic cellular automaton model was introduced [15]. In the present paper, we call this model the Drossel-Schwabl model or DS model for short. DS model is defined on a square lattice. Each site is occupied by a green tree, a burning tree, or is empty. During one time step, the system is parallel updated according to the following rules.

(1) A green tree without any burning nearest neighbors becomes a burning tree spontaneously with probability f (lightning).

(2) A green tree becomes a burning tree if at least one of its nearest neighbors is burning (fire spreading).

(3) A burning tree becomes an empty site.

(4) An empty site becomes a green tree with probability q (growing).

Periodic boundary conditions are assumed, and the initial configuration is a random configuration which consists of green trees and empty sites.

The system shows a critical state only in the double limit $q \to 0$ and $f/q \to 0$. In this limit the cluster-size distribution of green-tree sites follows a power law. We call this double limit the Drossel-Schwabl limit (DS limit).

DS limit represents a double separation of time scales: The time scale in which a green-tree cluster burns down is much shorter than the time scale in which an individual tree grows ($q \to 0$) and the latter time scale is much shorter than the time period of lightnings which occur at the same site ($f/q \to 0$). Such separation of time scales is quite frequent in nature, while the fire tuning of parameters to certain values only takes place accidentally in nature.

3. OUR MODEL

In the present paper, we propose a simple forest fire model in which elementary processes are the same as those in DS model, but the algorithm of procedures is different as explained below. Through the comparison of our model with DS model, we discuss the physical meaning of DS limit in this paper.

We consider a stochastic cellular automaton on a square lattice, which contains two parameters f (lightning probability) and q (growth probability of a tree at an empty site). Each site can be occupied by a green tree, a burning tree, or be empty. During one time step, the whole configuration is updated according to the following rules by turns.

(1) Every green site becomes a burning-tree site with probability f (lightning).

(2) Every fire spreads to each nearest neighbor sites. Each fire will continue to spread until it fails to spread to any unburnt sites.

(3) Every burning site becomes an empty site.

(4) Every empty site becomes a green site with probability q (growing).

Periodic boundary conditions are assumed, and the initial configuration is a random configuration which consists of green trees and empty sites. It should be noted that during fire spreading, the lightning and growing are prohibited. They can occur after any fire has ceased. In DS model we have to take DS limit to realize the separation of time scales among lighting, growing and fire spreading. In our model, however, their time scales are separated on algorithm. This setting is analogous to that in the sandpile models [16, 17], in which addition of particle (local perturbation) is taken place after any avalanche (spread of topplings) has ceased.

4. PERCOLATION TRANSITION OF GREEN-TREE SITES

We have studied our model by computer simulation. We regard a sequence of procedures (1) to (4) as one time step and investigate each configuration just before lightning (the procedure (1)). The system reaches a steady state after about 50 time steps. Figure 1 shows an example of steady configuration for $f = 0.1$, $q = 0.57$.

We regard the green-tree sites as open sites and the empty sites as closed sites and consider the site percolation problem [18] on steady configurations for each values of f and q. Here we consider the Neuman neighborhood and define open clusters. When there is a large cluster across the system, the system is considered to be the percolation state. The percolation probability θ of green-tree sites is approximated by [18]

$$\theta \simeq \text{average of } \frac{\text{largest cluster size}}{\text{system size}}. \qquad (1)$$

Figure 2 shows the q-dependence of θ for $f = 0.1$. A percolation transition is observed at a threshold growth-probability $q_c \simeq 0.57$. By changing the value of f, we evaluate each q_c and obtain a phase diagram. The result is shown in Fig.3(A). The percolation transition line is almost parallel to the f axis.

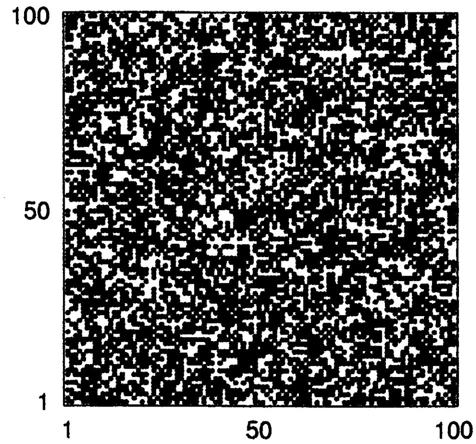


Fig. 1: An example of steady state of the present forest fire model for $f = 0.1$, $q = 0.57$. Green-tree sites are represented by black dots.

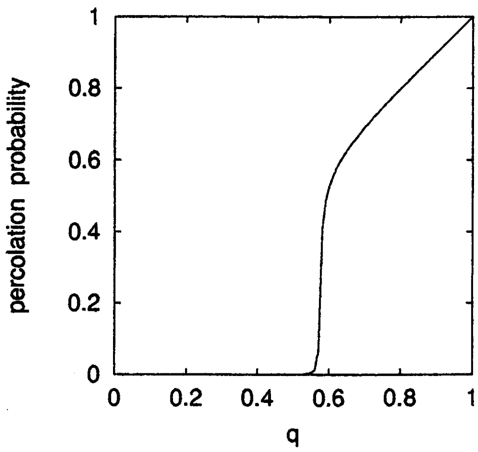
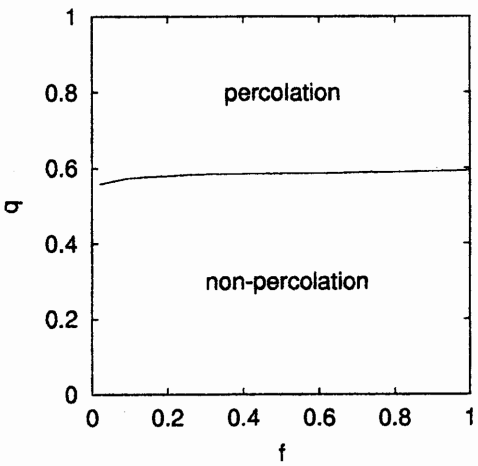


Fig. 2: Order parameter θ (the percolation probability of green-tree sites) versus q for $f = 0.1$. θ is approximated by eq.(1). The data are averaged over 120 time steps after discarding 50 time steps on a 2000×2000 square lattice with the periodic boundary condition. The threshold value is estimated as $q_c \simeq 0.57$.

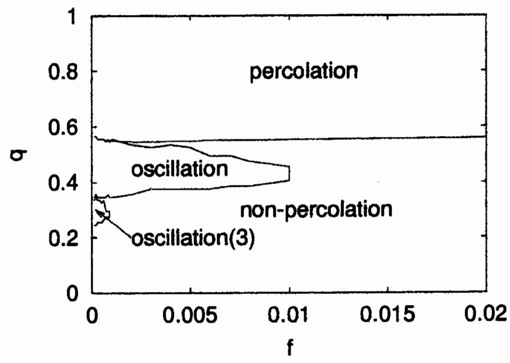
The reason why q_c is approximately independent of f is the following. If the system is in the percolation state, the large fire breaks out even by a little ignition, and after the large fire breaks out, almost all sites become empty. Thus the configuration before lightning is determined only by the growing process with probability q.

When $f = 1$, the observed percolation transition in our model is nothing but the well-known site percolation transition for the system in which each site is open (resp. closed) with probability q (resp. $1-q$) independently of each other sites. As we can see in Fig.3(A), as $f \to 1$, q_c becomes the value $0.5927\cdots$, which is the percolation threshold of the site percolation [18].

For large value of f, Our model may be essentially equivalent to the simple site percolation model. Now we study the phenomena found in the very small f region.



(A)



(B)

Fig. 3: **(A)** The phase diagram of the forest fire model. In the upper side of the transition line, $\theta > 0$. The transition line is almost parallel to f axis. **(B)** The phase diagram of the forest fire model in the region with very small f. There is the oscillation phase in the region for $f \le 0.02$. The oscillation(3) phase is the phase in which the system has positive probability to be in the percolation state once in every third step

5. OSCILATION PHASES

In the very small f region, we find new phase and the phases diagram is rather complicated. In the "oscillation" region in the phase diagram shown in Fig.3(B), the configuration oscillates between the percolation state ($\theta > 0$) and the non-percolation state ($\theta = 0$) every other step.

Figure 4 shows the percolation probability θ as a function of q for $f = 0.001$. The percolation threshold value is $q_c \simeq 0.25$ in this case. Between $q = 0.36$ and $q = 0.56$, θ is zero at the time steps t=even (resp. odd), while it has rather large values at t=odd (resp. even). Such oscillations disappear for $q < 0.36$ and for $q > 0.56$.

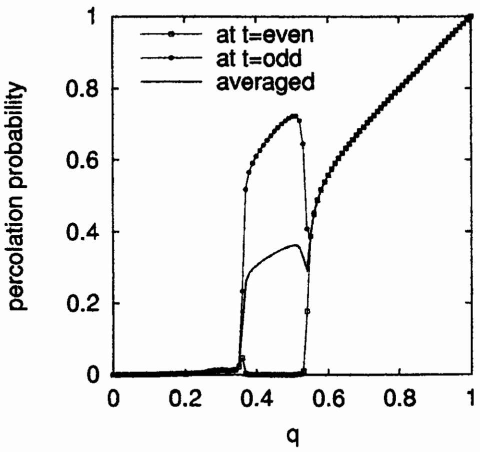


Fig. 4: Order parameter θ (the percolation probability) versus q for $f = 0.001$. These are averaged over 120 time steps after discarding 50 time steps on a 2000×2000 square lattice with the periodic boundary condition. The percolation threshold is $q_c \simeq 0.25$ in this case. The oscillation phase is found between $q = 0.36$ and $q = 0.56$.

The mechanism of the oscillation can be explained as follows. Since $q < 0.5927\cdots$ (the site-percolation threshold), the green-tree clusters are finite. It implies that the fire spreading caused by a lightning is local. When the ratio f/q is small, lightning and burning are very rare to occur compared to the growing of trees and thus the green tree clusters tend to become larger. For appropriate values of f and f/q, such situation will be repeated and at every other step the green-tree clusters have positive probability to cover the system (i.e. $\theta > 0$). If the system is covered by the green trees, a lightning causes a global fire spreading even if the lightning probability f is very small. Then $\theta = 0$ every other step, too.

In the region of smaller f, there is another oscillation phase in which $\theta > 0$ only at every third step. The parameter region of such a period-3 phase is indicated by oscillation(3) in Fig.3(B). We expect that a series of oscillation phases with longer periods $n \ge 4$ will be observed for much smaller f.

6. DISCUSSION AND FUTURE PROBLEM

As shown by Fig.5 schematically, a series of phase boundaries of oscillation phases enables us to draw an envelope. If we consider the situation in which the parameters q and f are changed along this envelope to take the $q \to 0$ limit, the system undergoes successive critical phenomena.

DS limit is given by the double limit, $q \to 0$ and $f/q \to 0$, in DS model. The envelope shown in Fig.5 for our model will give a physical realization of this double limit.

Critical phenomena are observed on each phase boundary of oscillation phase as well as on the usual percolation transition line in Fig.3(A). For example, cluster-size distribution of green-tree sites obeys a power-law. The critical exponents should be carefully evaluated for each oscillation phase. Dependence of critical phenomena on the period of oscillation n and comparison with the original DS model [19] will be challenging future problems.

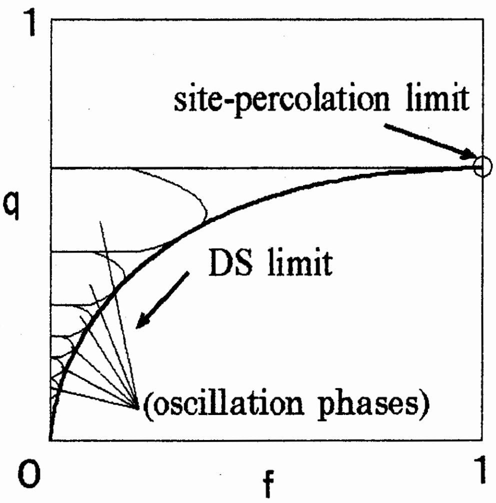


Fig. 5: A schematic phase diagram of our forest-fire model. The bold line is an envelope of a series of phase boundaries of oscillation phases. DS limit, $q \to 0$ and $f/q \to 0$, will be realized on this envelope.

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References

[1] A. H. Gentry, ed.: *Four Neotropical Rainforests* (Yale Univ. Press, New Haven, 1990).

[2] B. Walker and W. Steffen, ed.: *Global Change and Terrestrial Ecosystems* (Cambridge Univ. Press, Cambrige, 1996).

[3] C. H. Holling, G. Peterson, P. Marples, J. Sendzimir, K. Redford, L. Gunderson and D. Lambert: Self-organization in ecosystems: lumpy geometries, periodicities and morophologies, pp.346-384 in ref.2.

[4] R. V. Solé and S. C. Manrubia: Phys. Rev. E **51** (1995) 6250.

[5] R. V. Solé and S. C. Manrubia: J. Theor. Biol. **173** (1995) 31.

[6] S. C. Manrubia and R. V. Solé: Chaos, Solitons & Fractals **7** (1996) 523.

[7] M. Katori, S. Kizaki, Y. Terui, T. Kubo: Fractals **6** (1998) 81.

[8] S. Kizaki and M. Katori: J. Phys. Soc. Jpn. **68** (1999) 2553.

[9] S. P. Hubbell and R. B. Foster: *Plant Ecology*, ed. M. J. Crawley (Blackwell, Oxford, 1986) p. 77.

[10] T. Kubo, Y. Iwasa and N.Furumoto: J. Theor. Biol. **180** (1996) 229.

[11] H. Tanaka and T. Nakashizuka: Ecology **78** (1997) 612.

[12] P. Bak: *How Nature Works*, (Oxford Univ. Press, Oxford, 1997).

[13] H. J. Jensen: *Self-Organized Criticality* (Cambridge Univ. Press, Cambridge, 1998).

[14] P. Bak, K. Chen and C. Tang: Phys. Lett. **A 147** (1990) 297.

[15] B. Drossel and F. Schwabl: Phys. Rev. Lett. **69** (1992) 1629.

[16] P. Bak, C. Tang and K. Wiesenfeld: Phys. Rev. Lett. **59** (1987) 381.

[17] A. Vespignani and S. Zapperi: Phys. Rev. E **57** (1998) 6345.

[18] D. Stauffer and A. Aharony: *Introduction to Percolation Theory*, Second Edition, (Taylor & Francis, London, 1992) pp.52.

[19] S. Clar, B. Drossel and F. Schwabl: Phys. Rev. E **50** (1994) 1009.

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