Workshop on
“Recent Topics of Statistical Mechanics and Probability Theory ”

DATE: 29 March 2012

PLACE: Chuo University, Korakuen Campus, Faculty of Science and Engineering, Building No.3, Room 3300 (3rd floor)

PROGRAMME:

13:00–13:30 Makoto KATORI (Chuo):
Vicious Brownian motion, O’Connell process, and equilibrium Toda lattice
(10 min. break)

13:40–14:10 Hideki TANEMURA (Chiba):
Stochastic differential equations related to soft-edge scaling limit
(10 min. break)

14:20–14:50 Sergio ANDRAUS (Tokyo):
Dyson’s Brownian motion model as a special case of Dunkl processes and Dunkl’s intertwining operators
(20 min. break)

15:10–15:40 Tomohiro SASAMOTO (Chiba):
Stationary two-point correlation for the KPZ equation
(10 min. break)

15:50–16:20 Takashi IMAMURA (Tokyo):
Replica analysis of the stationary 1d KPZ equation
(20 min. break)

16:40-17:40 Piotr GRACZYK (Angers):
A multidimensional Yamada-Watanabe theorem with applications to particle systems and matrix SDEs

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ABSTRACTS

Makoto KATORI (Chuo University)
Vicious Brownian motion, O'Connell process, and equilibrium Toda lattice

Vicious Brownian motion is a system of Brownian particles in one dimension such that if two particles collide they kill each other. When this system is considered under noncolliding condition, it becomes equivalent to Dyson's GUE Brownian motion model, which is the eigenvalue process of the Hermitian-matrix-valued Brownian motion, known as a typical example of log-gases. We study a softened version of vicious Brownian motion in which we allow particles not only to meet neighbors but also to change order of positions in one dimension (letting them be more friendly), although in such situations survival probability of particles decays exponentially. Recently O'Connell introduced an interacting diffusive particle system in order to study a directed polymer model in finite temperatures on 1+1 dimensions. The infinitesimal generator of O'Connell's process is a harmonic transform of the quantum Toda-lattice Hamiltonian by the Whittaker function. As a physical interpretation of this construction, we show that O'Connell's process (without drift) is realized as the softened version of vicious Brownian motion conditioned that all particles survive forever. We would like to demonstrate also that the initial-configuration problem of the one-dimensional O'Connell process is mapped to the boundary-condition problem of the Gibbs state of Toda lattice in a two-dimensional space (which is frozen to be the Gelfand-Tsetlin polytope at the zero temperature).

Hideki TANEMURA (Chiba University)
Stochastic differential equations related to soft-edge scaling limit

Consider the eigenvalues of Gaussian unitary ensemble, **GUE** (Gaussian orthogonal ensemble, **GOE**, and Gaussian symplectic ensemble, **GSE**, respectively). In *the bulk scaling limit*, the limit distribution is the determinantal point process $\mu_{\sin,2}$ with the sine kernel (the quaternion determinantal point processes $\mu_{\sin,1}$ and $\mu_{\sin,4}$, respectively). Osada [1] [2] constructed an infinite particle system represented by a diffusion process which has $\mu_{\sin,\beta}$ as a reversible measure by the Dirichlet form approach, and proved that the system satisfies the following infinite dimensional stochastic differential equation (ISDE) :

$$dX_j(t) = dB_j(t) + \frac{\beta}{2} \sum_{k \in \mathbf{N}, k \neq j} \frac{dt}{X_j(t) - X_k(t)}, \quad j \in \mathbf{N}, \quad t \in [0, \infty),$$

for $\beta = 1, 2$ and 4, where $B_j(t)$'s are independent one-dimensional standard Brownian motions, and $\mathbf{N} = \{1, 2, \dots\}$.

In *the soft-edge scaling limit*, the limit distribution is the determinantal point process $\mu_{\mathrm{Ai},2}$ with the Airy kernel (the quaternion determinantal point processes $\mu_{\mathrm{Ai},1}$ and $\mu_{\mathrm{Ai},4}$, respectively). In this talk we construct an infinite particle system which has $\mu_{\mathrm{Ai},\beta}$ as a reversible measures and show that it solves ISDE:

$$dX_j(t) = dB_j(t) + \lim_{L \to \infty} \left\{ \frac{\beta}{2} \sum_{k \neq j, |X_k(t)| < L} \frac{1}{X_j(t) - X_k(t)} - \int_{|x| < L} \frac{\widehat{\rho}(x) dx}{-x} \right\} dt,$$

$j \in \mathbf{N}$, $t \in [0,\infty)$, with $\widehat{\rho}(x) = \frac{1}{\pi}\sqrt{-x}\mathbf{1}(x<0)$, for $\beta = 1, 2$ and 4.

This is a joint work with Hirofumi Osada (Kyushu University).

[1] Osada, H. : Interacting Brownian motions in infinite dimensions with logarithmic interaction potentials. to appear in Ann. of Probab.

[2] Osada, H. : Infinite-dimensional stochastic differential equations related to random matrices. to appear in Probab. Theory Relat. Fields.

Sergio ANDRAUS (University of Tokyo)

Dyson's Brownian motion model as a special case of Dunkl processes and Dunkl's intertwining operators

Dyson's Brownian motion model is a family of systems in which N Brownian particles interact repulsively through a log-potential in one dimension, and it is indexed by the positive real parameter β. Dunkl processes are stochastic processes defined as a generalization of N-dimensional Brownian motion based on a set of differential-difference operators, called Dunkl operators. These operators depend on the choice of a finite set of vectors called root system. When the A-type root system is chosen, its associated Dunkl process describes a system of Brownian particles that repel each other in one dimension and exchange positions spontaneously. An important part of the results from the theory of Dunkl operators is obtained through the use of the intertwining operator. This operator relates partial derivatives and Dunkl operators, but its explicit form is unknown in general.

We show that the A-type Dunkl process of parameter k equal to $\beta/2$ under a symmetric initial condition is equivalent to Dyson's model. From this equivalence, we extract an expression for the effect of the intertwining operator on symmetric polynomials. We show that in the $k \to \infty$ limit it maps symmetric functions into a function of the sum of their variables. This allows us to study the zero-temperature limit of Dyson's model, and we show that the final configuration is proportional to a vector of the roots of the Hermite polynomials multiplied by the square root of the process time, while being independent of the initial configuration. We conclude by briefly discussing two topics of further study on the intertwining operator: its symmetric eigenfunctions and its effect on non-symmetric polynomials.

Tomohiro SASAMOTO (Chiba University)

Stationary two-point correlation for the KPZ equation

For the last few years, there have been renewed interests in the one-dimensional KPZ equation. In particular the height distributions of the interface have been computed for a few cases such as the narrow wedge initial condition.

In this talk, after reviewing the recent developments, we present explicit formulas of the height distribution and the two point correlation function for the KPZ equation in its stationary regime, which is one of the most natural and important case of the problem [1]. We explain basic ideas for the derivation based on replica method and discuss its importance from the point of view of statistical mechanics. The details of the derivation are shown in the next presentation by Takashi Imamura [2].

[1] T. Imamura, T. Sasamoto, arXiv:1111.4634

[2] T. Imamura, next presentation in this workshop

Takashi IMAMURA (University of Tokyo)

Replica analysis of the stationary 1d KPZ equation

I present our recent result (arXiv:1111.4634) on the height distribution and two-point correlations of the 1d KPZ equation under stationary situation focusing on the details of the derivation. The analysis is based on the replica approach in which the Bethe ansatz results on the 1d attractive delta Bose gas can be available. Furthermore to deal with the stationary state, we introduce some key ideas (the two-sided Brownian motion initial condition, a new combinatorial identity, etc). This is a joint work with Tomohiro Sasamoto. Before my talk, he talks about basic ideas of the derivation and (statistical mechanical) meanings of our result .

Piotr GRACZYK (LAREMA, Universite D'Angeres)

A multidimensional Yamada-Watanabe theorem with applications to particle systems and matrix SDEs

In a common paper with J.Malecki we prove a new version of multidimensional and a matrix Yamada-Watanabe theorems. They have motivations and useful applications to matrix SDEs for matrix processes of squared Bessel type, like Wishart or Jacobi processes. Their eigenvalues processes are also concerned by our results. They are non-colliding particle systems of BESQ processes, important in mathematical and statistical physics.